Formalization of an Ideology

Patrick Schäfer

University of Cologne

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Overview

Motivation

- This is my master thesis in economics
- Economics uses mathematics to formulate its theories
  - Idea of how the economy works $\rightarrow$ Formalization $\rightarrow$ Empirical critique
- Key advantages of formalization:
  - transparency of assumptions
  - unambiguous conclusions
  - empirical testing and falsification
- Define ideologies as coherent worldviews
- Formalization allows testing and structured scientific discourse

Why Social-Darwinism

- Analytical suitability
  - Conceptually simple
  - Mostly coherent
- Clarity of source
  - Authoritative text: *Mein Kampf*
  - Fixed in time, not fluid
  - Ideas were put into practice
- Relevance
  - Little direct comparison with the evidence
    - A lot of literature on contextualizing the ideology
    - A lot of science on refuting its assumptions implicitly
  - Retains an undeserved mystique (Hitler, Hartmann, et al., 2016)

ℕaproche

- ℕaproche (Natural Proof Checking) is a proof assistant
- It processes a controlled fragment of natural mathematical language
  - supports some standard grammatical structures
  - supports some symbolic notation
  - supports definition-theorem-proof format
- Translation into first-order logic tasks
  - handled by built-in reasoning modules
  - or by external automated theorem provers
- Natural reading of code is a precondition for using formalization in economics
- Can bring advantages of formalization to subjects outside of mathematics
  - elimination of ambiguity
  - enhanced verifiability
  - enabling large-scale collaborations, etc.

Two Pages of the Formalization

I Theorems: Rise of Nations

Mein Kampf ends by stating that a state that would put its highest quality people in positions of influence, would "with almost mathematical certainty" bound to dominate the world (p. 1619f[223]). The following theorem shows that if the average quality of a nation remain high, its culture goes against infinity and in an environment with otherwise low average quality has a share converging to one.

[read examples/darwinism/35lemmata.ftl.tex][check off][prove off]

Let W denote a world. Let t denote a point in time.

Definition 358. Let $n \in N^W$. n is continually high quality in W iff $Q_n^W(t) = R^W(5)$ for all points in time t.

If a nation continues to have a high quality level, then the culture level goes against infinity (p. 767[224]).

Proposition 359. Let $n \in N^W$. Assume n is continually high quality in W. Then

$$\lim_{x \to \infty} C_n^W(x) = \infty.$$

Proof. C_n^W is a differentiable real map.

For all $t \in [0, \infty)$ $Q_n^W(t) \geq R^W(4)$ and $\partial C_n^W(t) \geq \eta^W$ (by def continually high quality, lemma change in culture 1). $\lim_{x \to \infty} C_n^W(x) = \infty$ (by lemma 8). ■

In the following theorem it is shown that a high quality nation in a low quality environment will dominate long-term, i.e. its share converges against one. The proof of the theorem has to steps: first, it has to be established that the its share ($S_1^W(t)$) converges at all; and second that this point of convergence is one. To prove the convergence, we define $\psi(t) := F_1^W(t) - F_2^W(t)$ and in (A) show that if $\psi(t) < 0$ then $\partial\psi(t) \geq \mu$ for some positive μ. This allows us to employ two lemmata saying that there exists a point in time τ such that $\psi(\tau) > 0$ and for all $t \geq \tau$ $\psi(t) \geq 0$. Since we assume the, world is practically bipolar, we can reformulate the

share development axiom to

$$\partial S_1^W(t) = (\frac{m-1}{m} \cdot (F_1^W(t) - F_2^W(t))) \cdot S_1^W(t).$$

Thus $S_1^W(t)$ is increasing for all $t \geq \tau$. Since S_1^W is also bounded above by one, S_1^W converges.

Next, it is to show that S_1^W's convergence point p is equal to one. Trivially $p \in (0, 1]$, as $S_1^W(\tau) > 0$. Thus it is left to show that $p \notin (0, 1)$. Assume for the purpose of contradiction that $p \in (0, 1)$, we can find a point in time τ', for which $\partial S_1^W(t) \geq \nu$ for some positive real number ν, assuming that S_1^W converges against p. But then $S_1^W(t) \to \infty$, which contradicts that $S_1^W(t) \to p$. Therefore $p \notin (0, 1)$ and $p = 1$.

Theorem 360 (Rise of Nations). Assume W is mostly low quality. Assume 1 is continually high quality in W. Then

$$\lim_{x \to \infty} S_1^W(x) = 1.$$

Proof. Take $m = |N^W|$. Take $s = S_1^W$. Take $qL = R^W(1)$. Take $qH = R^W(5)$. (0) Take $\psi = (F_1^W + (-F_2^W))$.

(2) ψ is a differentiable real map (by minus derivative 2, derivative addition rule 2). Indeed F_1^W, F_2^W are differentiable real maps (by signature fitness development, lemma fitness development real map).

(4) s is a differentiable real map (by share development of nation is real map,ax share development).

Take $\mu = (\eta^W)^2$. (3) μ is a positive real number and 0 is a real number.

(A) ψ is satisfying nonnegativity condition larger μ since 0 (by def nonnegativity condition). Indeed we can show that for all $t \in [0, \infty)$ if $\psi(t) < 0$ then $\partial\psi(t) \geq \mu$. Proof. Let $t \in [0, \infty)$. Assume $\psi(t) < 0$.

(1) $\partial Q_1^W(t) = 0$. Proof. (1A) $Q_1^W = \mathfrak{f}_{0\_+R^W(5)}$. Proof. Let t' be a point in time. $R^W(5). = \mathfrak{f}_{0\_+R^W(5)}(t')$ (by linear lem 1). $Q_1^W(t') = R^W(5) = \mathfrak{f}_{0\_+R^W(5)}(t')$. End.

Structure of the Thesis

- Introduction
  - Summary and structure of the paper
  - Motivation and challenges
  - Introduction to $\mathbb{N}$aproche
- Ideological content of *Mein Kampf*
- Formal model
  - Assumptions from the text
  - Formalization process
- Strengths and weaknesses of the model
- Application: Hitler's foreign policy prediction
- Critique of assumptions
- Conclusion

The Ideology

"As [a **high quality** */ Aryan] conqueror subjugated the inferior peoples, and regulated their practical activity under his orders, according to his will, and for his own purposes[. . . , he became] the preserver and encourager of* **civilization***. For this depended entirely upon his abilities, and thus on his survival. [. . . With time] the Aryan surrendered the purity of his blood, and thus [. . .] went down in the* **mixture of races***, and gradually* **lost more and more of his cultural capacities** *until finally he began to resemble the aborigine more than his own forefathers [. . .] For a time he could still live upon existing cultural substance, but then ossification set in, and finally oblivion claimed him.*

Thus civilizations and empires **collapse** *to make way for new structures." (Hitler and Unknown, 1939, p. 288)*

Model: Constituents

A world $W = (N^W, R^W, D^W, I^W, Q^W, C^W, F^W, S^W, \eta^W)$ has

- nations $N^W = \{1, \ldots, n\}$,
- racial categorization $R^W \in [0,1]^5$ with $R^W(i) < R^W(i+1)$.
- demographies $D^W : N^W \to (T \to [0,1]^3)$ with $I^{D_n^W}(t) + m^{D_n^W}(t) + h^{D_n^W}(t) = 1$,
- influence functions $I^W : N^W \to (T \to ([0,1] \to \mathbb{R}_{\geq 0}))$,
- average qualities $Q^W : N^W \to (T \to [0,1])$,
- cultural levels $C^W : N^W \to (T \to \mathbb{R}_{\geq 0})$,
- fitness levels $F^W : N^W \to (T \to \mathbb{R}_{\geq 0})$,
- shares $S^W : N^W \to (T \to [0,1])$ with $\sum_{n=1}^{|N^W|} S_n^W(t) = 1$.

$\eta^W > 0$ is a technical parameter.

Model: Axioms

- Change of demography:

$$\begin{aligned}\partial l^D(t) &= l^D(t) \cdot (l^D(t) - 1 + 2m^D(t)),\\ \partial m^D(t) &= m^D(t) \cdot (m^D(t) - 1 + 2h^D(t)) + 2h^D(t) l^D(t),\\ \partial h^D(t) &= h^D(t) \cdot (h^D(t) - 1).\end{aligned}$$

- Change of culture $\partial C_n^W(t) = fc^W(C_n^W(t), Q_n^W(t))$:

$$\begin{aligned} Q_n^W(t) \geq R^W(4) &\implies \partial C_n^W(t) \geq \eta^W,\\ \left[\forall t' \in [t, \infty) : \ Q_n^W(t') \leq R^W(2)\right] &\implies \lim_{x \to \infty} C_n^W(x) = 0.\end{aligned}$$

- Change of fitness $F_n^W(t) = ff^W((C_n^W(t), Q_n^W(t), S_n^W(t)))$:

$$D_1 ff^W(t) \geq \eta^W, \ D_2 ff^W(t) > 0, \text{ and } D_3 ff^W(t) < 0.$$

- Share development:

$$\partial S_n^W(t) = (F_n^W(t) - \bar{F}^W(t)) \cdot S_n^W(t).$$

Model: Theorems

Theorem (Rise of Nations)

Assume W is mostly low quality. Assume 1 is continually high quality in W. Then

$$\lim_{x \to \infty} S_1^W(x) = 1.$$

Theorem (Summary Theorem Fall)

Assume W is mostly low quality. Assume 1 is nondiscriminating in W. Assume $I^{D_1^W}(0) > 0$. Then

$$\lim_{x \to \infty} S_1^W(x) = \frac{1}{|N^W|}.$$

Strength and Weaknesses of the Model

Hitler saw the mission of humanity in increasing culture (Hitler and Unknown, 1939, p. 389).

- Abolition of democracy
- Genocide of Jews
- World conquest

What is not modelled:

- No interior politics
- Specific foreign policy
- Development of new races ("Microfoundation")

Critique

- Race is not a biological concept:
  - Only one species: *homo sapiens*.
  - "99.8% of whole genome is identical between two [randomly-chosen] individuals" (B.-J. Kim, Choi, and S.-H. Kim, 2023, p. 11)
  - The "majority of human genetic diversity exists within local populations (85%), with much less among local populations (5%) or between major geographic regions or 'races' (10%)" (Relethford, 2002, p. 1)
- That self-sacrifice is independent of race (Whitehouse, 2018)
- Failed prediction in *Mein Kampf*
  - Collapse of Russia (Hitler and Unknown, 1939, p. 636)
  - Rise of Germany (Hitler and Unknown, 1939, Afterword)
  - Impossibility of a Jewish state (Hitler and Unknown, 1939, p. 294)
- Hitler has no concept of human dignity and thus human rights

Thank you very much!

Questions?

References

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