Why We Need Structured Proofs in Mathematics

Mauricio Ayala-Rincón
ayala@unb.br

Gabriel Ferreira Silva
gabrielfsilva1995@gmail.com

Departments of Computer Science and Mathematics, Universidade de Brasília

When mathematicians write a standard proof of some theorem, they have to decide on which level of detail they will present their argument. If they provide too little justification, readers may spend a lot of time filling the holes left or, even worse, may not understand why a specific step of the proof is (or not) correct. However, more argumentation is not necessarily always better, since this may obscure the "big picture" and some readers may be more interested in seeing the "big picture" than in checking every tiny detail of the proof. Therefore, the ideal level of detail in a mathematical proof varies from reader to reader, as it depends on the reader's previous knowledge and the reader's intention.

The standard way of writing mathematical proofs, which will henceforth be called prose proofs, cannot satisfy everyone. In contrast with prose proofs, we have the concept of structured proofs, proposed by Leslie Lamport in [3] and described in [4, 3] and also in [5]. In a structured proof, the steps and substeps are hierarchically presented. The proof is decomposed as a tree of steps, and each step can be further decomposed into smaller subtrees of substeps and so on. By correctly numerating each step and by using the right indentation, the hierarchy of the proof is clear at first sight. Examples of structured proofs can be found in the extended version of this paper, available at `http://ayala.mat.unb.br/publications.html`.

If writing structured proofs is accompanied by the discipline of explaining every step in meticulous detail, the probability of obtaining an incorrect proof or a wrong result diminishes. That is because the hierarchy of the proof will allow the writer to add justifications to a given step without "clouding" other steps of the proof, and also make it easier to check if all the corner cases were handled.

Since structured proofs can make it harder to obtain an incorrect result, a comparison may arise between structured proofs and interactive theorem provers (ITPs). By using an ITP, the chances of obtaining an incorrect result are significantly lower than by using a structured proof, as the computer checks every step of the proof. However, structured proofs also have advantages over ITPs, as they are faster to write and more readable. As members of a research group bridging interest from graduate programs in Mathematics and Computer Science we also have a huge interest in motivating mathematicians to use ITPs. For this we have developed mathematical theories in PVS as well as short-courses to attract the interest of such audience [2, 1].

References

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In: A. Editor, B. Coeditor (eds.): Proceedings of the XYZ Workshop, Location, Country, DD-MMM-YYYY, published at http://ceur-ws.org