System Description: XSL-based Translator of Mizar to LaTeX

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A New Version of the Mizar to LaTeX Translator

- Re-implemented as XSL stylesheets instead of as Object Pascal programs
  - TeX translation initiated in 1989 by A. Trybulec (†2013) and P. Rudnicki (†2012)
    - A Collection of TeX-ed Mizar Abstracts
    - *Formalized Mathematics* journal established in 1990
  - Project taken over in 1991 by G. Bancerek (†2017)
    - On-line *Journal of Formalized Mathematics* initiated in 1996 (discontinued since 2005)
    - Hyperlinked representation of the Mizar Mathematical Library (reimplemented by J. Urban in 2005)
- Current translation can be used to generate both LaTeX/PDF and HTML/MathJax code
- Experimenting with generation of full proofs
- Available on-line and through the Mizar Emacs interface

Formalized Mathematics - Example Recent Publication

FORMALIZED MATHEMATICS
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degruyter.com/view/j/forma

Klein-Beltrami Model. Part I

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Summary. Tim Makarios (with Isabelle/HOL[1]) and John Harrison (with HOL-Light[2]) shown that "the Klein-Beltrami model of the hyperbolic plane satisfy all of Tarski's axioms except his Euclidean axiom" [3], [4], [14], [5].

With the Mizar system [2], [7] we use some ideas are taken from Tim Makarios' MSc thesis [13] for the formalization of some definitions (like the absolute) and lemmas necessary for the verification of the independence of the parallel postulate. This work can be also treated as further development of Tarski's geometry in the formal setting [6]. Note that the model presented here, may also be called "Beltrami-Klein Model", "Klein disk model", and the "Cayley-Klein model" [1].

MSC: 51A05 51M10 03B35

Keywords: Tarski's geometry axioms; foundations of geometry; Klein-Beltrami model

MML identifier: BKMODEL1, version: 8.1.07 5.47.1318

1. Preliminaries

From now on a, b, c, d, e, f denote real numbers, g denotes a positive real number, x, y denote complex numbers, S, T denote elements of $\mathcal{R}^2$, and u, v, w denote elements of $\mathcal{E}^3_{\mathrm{T}}$.

Now we state the propositions:

(1) Let us consider elements P_1, P_2, P_3 of the projective space over $\mathcal{E}^3_{\mathrm{T}}$. Suppose u is not zero and v is not zero and w is not zero and $P_1 =$ the direction of u and $P_2 =$ the direction of v and $P_3 =$ the direction of w. Then P_1, P_2 and P_3 are collinear if and only if $\langle|u, v, w|\rangle = 0$.

[1] https://www.isa-afp.org/entries/Tarskis_Geometry.html
[2] https://github.com/jrh13/hol-light/blob/master/100/independence.ml

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(2) If ($a \neq 0$ or $b \neq 0$) and $a \cdot d = b \cdot c$, then there exists e such that $c = e \cdot a$ and $d = e \cdot b$.

(3) If $a^2 + b^2 = 1$ and $(c \cdot a)^2 + (c \cdot b)^2 = 1$, then $c = 1$ or $c = -1$.

(4) $a \cdot u + (-a) \cdot u = 0_{\mathcal{E}^3_{\mathrm{T}}}$.

(5) If $0 \leqslant a$ and $c < 0$ and $\Delta(a, b, c) = 0$, then $a = 0$.
Proof: $0 \leqslant b^2$. □

(6) $\sum({}^2(T - S)) = \sum({}^2(S - T))$.

(7) If $a^2 + b^2 = 1$ and $c^2 + d^2 = 1$ and $c \cdot a + d \cdot b = 1$, then $b \cdot c = a \cdot d$.

(8) If $a^2 + b^2 = 1$ and $a = 0$, then $b = 1$ or $b = -1$.

(9) $0 \leqslant a^2$.

(10) If $(a \cdot b)^2 + b^2 = 1$, then $b = \frac{1}{\sqrt{1+a^2}}$ or $b = \frac{-1}{\sqrt{1+a^2}}$.

(11) If $a \neq 0$ and $b^2 = 1 + a \cdot a$, then $a \cdot \frac{1}{b} \cdot a \cdot \frac{-1}{b} + \frac{1}{b} \cdot \frac{-1}{b} = -1$.
Proof: $b \neq 0$. □

(12) $a^2 \cdot \frac{1}{b^2} = (\frac{a}{b})^2$.

(13) $a^2 + b^2 = 1$ if and only if $[a, b] \in \mathrm{circle}(0, 0, 1)$.

(14) $a^2 + b^2 = g^2$ if and only if $[a, b] \in \mathrm{circle}(0, 0, g)$.

(15) If $a \neq 0$ and $-1 < a < 1$ and $b = \frac{2+\sqrt{\Delta(a \cdot a, -2, 1)}}{2 \cdot a \cdot a}$, then $(1 + a \cdot a) \cdot b \cdot b - 2 \cdot b + 1 - b \cdot b = 0$.
Proof: $0 \leqslant 1 - a^2$. $\Delta(a \cdot a, -2, 1) \geqslant 0$. □

(16) Suppose $a^2 + b^2 = 1$ and $-1 < c < 1$. Then there exists d and there exists e and there exists f such that $e = d \cdot c \cdot a + (1 - d) \cdot (-b)$ and $f = d \cdot c \cdot b + (1 - d) \cdot a$ and $e^2 + f^2 = d^2$.

(17) If $a^2 + b^2 < 1$ and $c^2 + d^2 = 1$, then $(\frac{a+c}{2})^2 + (\frac{b+d}{2})^2 < 1$.

(18) If $|S|^2 \leqslant 1$, then $0 \leqslant \Delta(\sum({}^2(T - S)), b, \sum({}^2S) - 1)$.

(19) If $a^2 + b^2$ is negative, then $a = 0$ and $b = 0$.

(20) If $u = [a, b, 1]$ and $v = [c, d, 1]$ and $w = [\frac{a+c}{2}, \frac{b+d}{2}, 1]$, then $\langle|u, v, w|\rangle = 0$.

(21) (i) $a \cdot |(u, v)| = |(a \cdot u, v)|$, and
(ii) $a \cdot |(u, v)| = |(u, a \cdot v)|$.

In the sequel a, b, c denote elements of $\mathbb{R}_{\mathrm{F}}$ and M, N denote square matrices over $\mathbb{R}_{\mathrm{F}}$ of dimension 3.

Now we state the propositions:

(22) If $M = \mathrm{symmetric3}(0, 0, 0, 0, 0, 0)$, then $\mathrm{Det}\, M = 0_{\mathbb{R}_{\mathrm{F}}}$.

(23) Suppose $N = \langle\langle a, 0, 0\rangle, \langle 0, b, 0\rangle, \langle 0, 0, c\rangle\rangle$. Then
(i) $M^{\mathrm{T}} \cdot (N \cdot M)_{1,1} = a \cdot (M_{1,1}) \cdot (M_{1,1}) + b \cdot (M_{2,1}) \cdot (M_{2,1}) + c \cdot (M_{3,1}) \cdot (M_{3,1})$, and

Description of the New Technology

- XSL translation templates are applied to XML representation
  - Weakly-strict Mizar (*.wsx file)
  - Semantic representation generated by the Mizar verifier (*.xml file) is used to decode links to external articles
- Bibliographic metadata (*.bib file) is also translated to a special XML format and merged with information extracted from the Mizar article

Example Stylesheet in Action: recognize-programs.xsl

```
theorem
  for n,s,i being Variable of g st ex d being Function st d.n = 1 & d.s
  = 2 & d.i = 3 & d.b = 4 holds s:=1\;for-do(i:=2, i leq n, i+=1, s*=i)
  is_terminating_wrt g
```

Now let Γ denotes the program

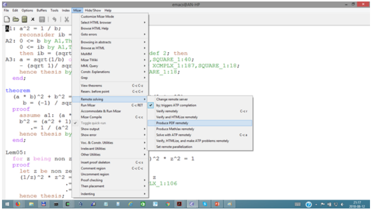
s:=1;
for i:=2 until $i\,\mathrm{leq}\,n$ step $i+=1$ do
  $s*=i$
done

Then we state the propositions:

(56) Let us consider variables n, s, i in g. Suppose there exists a function d such that $d(n) = 1$ and $d(s) = 2$ and $d(i) = 3$ and $d(b) = 4$. Then Γ is terminating w.r.t. g.

Remote Service Accessible via Emacs

- The service is similar to the MizAR ATP service for Mizar
- Sending articles from Emacs to the service for PDF and HTML translation of their current work independently of the publication process of *Formalized Mathematics*
  - *Mizar→Remote solving→Produce PDF online*
  - *Mizar→Remote solving→Produce MathJax online*
- `mizar-tex-remote` function in the current `mizar.el` Emacs Lisp mode for Mizar



Future Work Directions

- Focus on adding more natural language based linguistic features to the generated mathematical text
- Human-friendly presentation of the structure of (nested with varying levels of importance) proofs
- Integration with various user interfaces
- Streamlining the publication process of *Formalized Mathematics* papers