Defining futures

Formal Dependability Modeling and Analysis: A Survey

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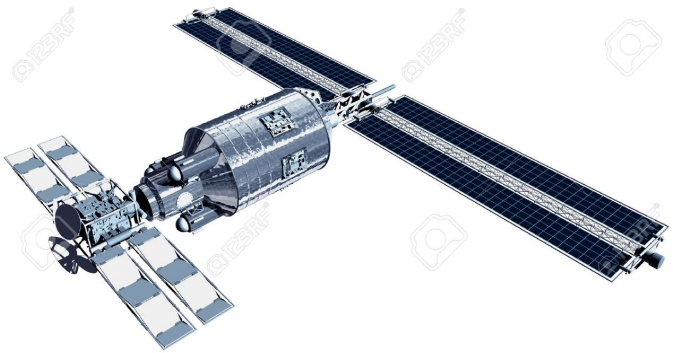
CICM 2016
Bailystock, Poland

July 27, 2016

Outline

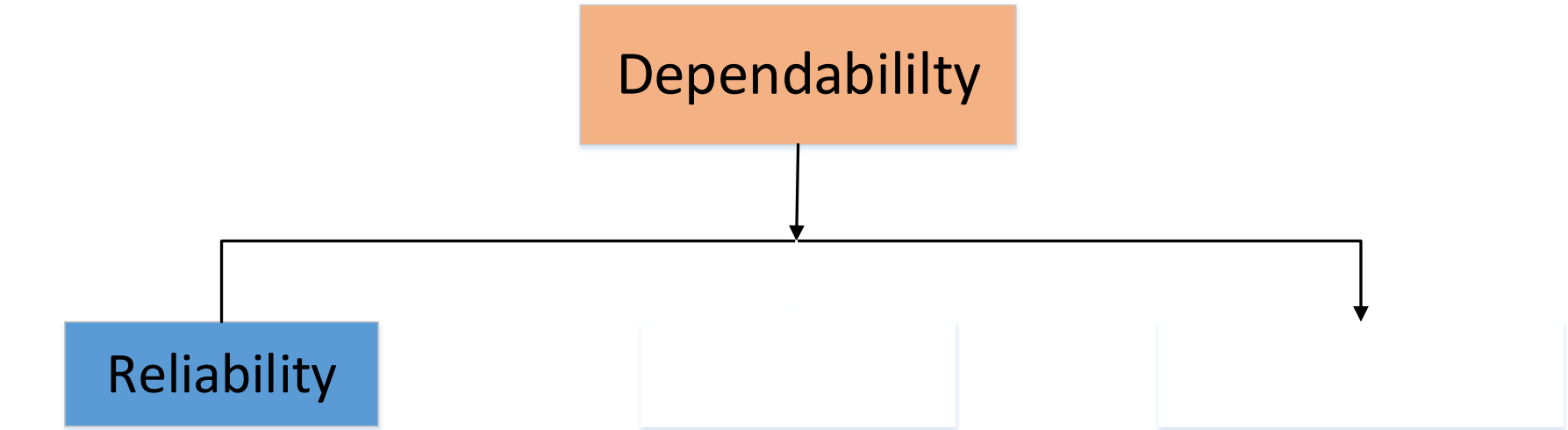
Dependability

Safety-critical Systems



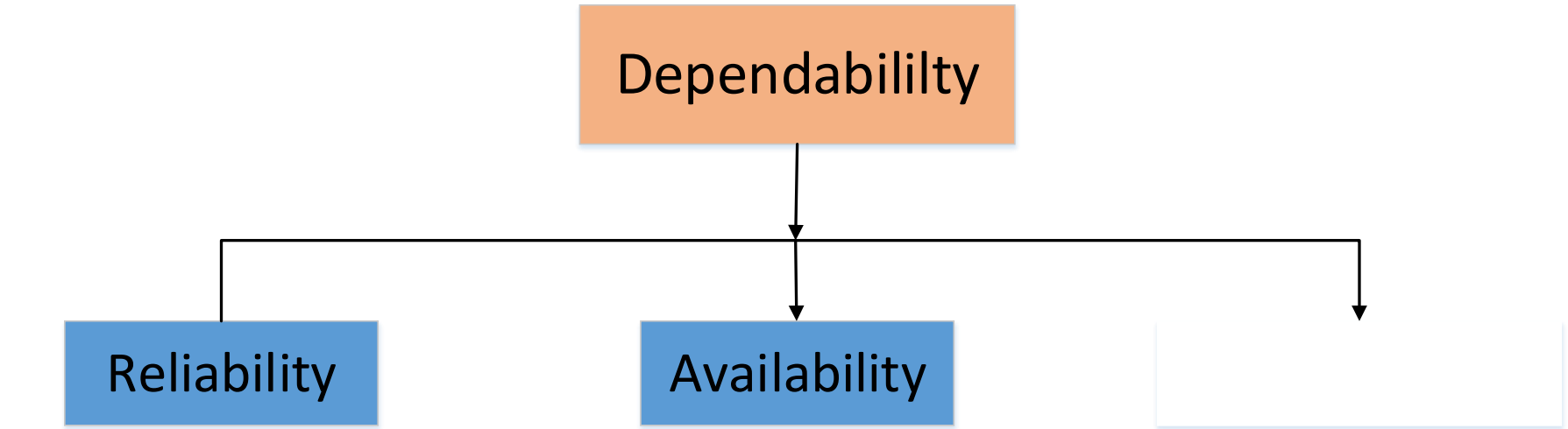
- More stringent dependability requirements
  - Main motivation for Formal Dependabiltiy Modeling and Analysis

Dependability



The ability of system to deliver services as specified within a given time

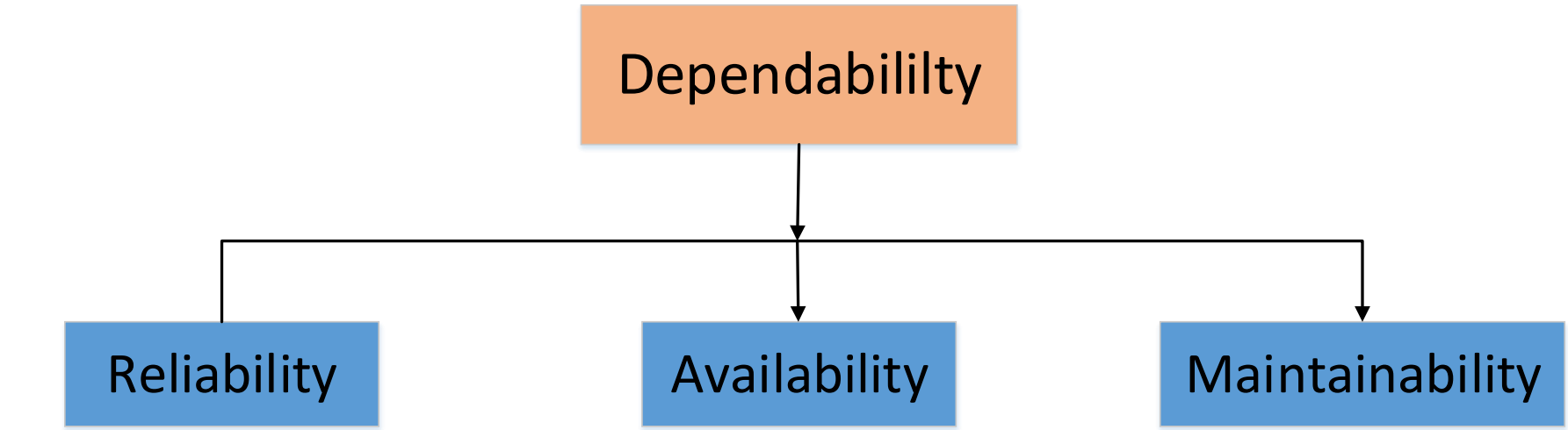
Dependability



The ability of system to deliver services as specified within a given time

The ability of system to deliver services when required

Dependability



The ability of system to deliver services as specified within a given time

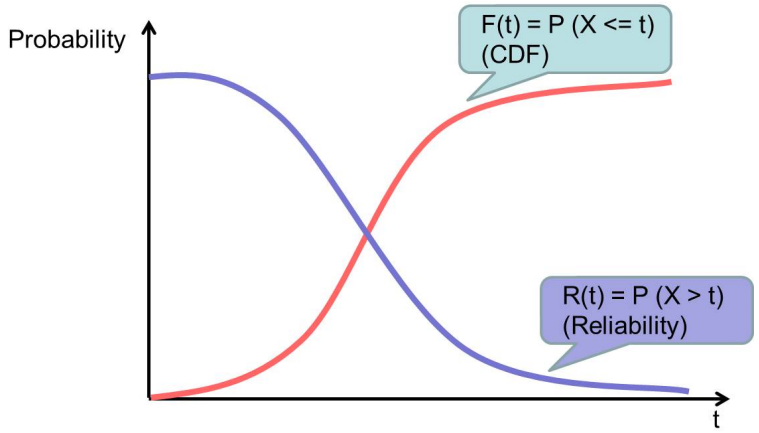
The ability of system to deliver services when required

The ability of a system to restore to operational status after a failure occurs

Formal Definitions

- Reliability $= \mathbb{P}$(no failure occurs before certain time)

$$
\begin{aligned}
R(t) &= Pr(X > t) \\
&= 1 - Pr(X \leq t) \\
&= 1 - F_X(t)
\end{aligned}
$$



Formal Definitions

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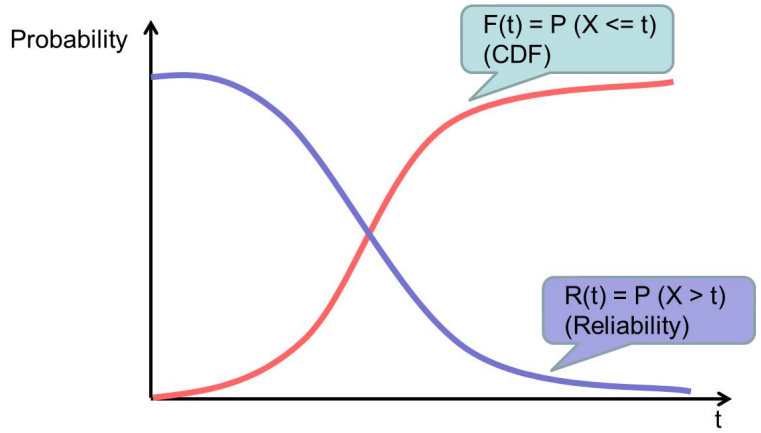
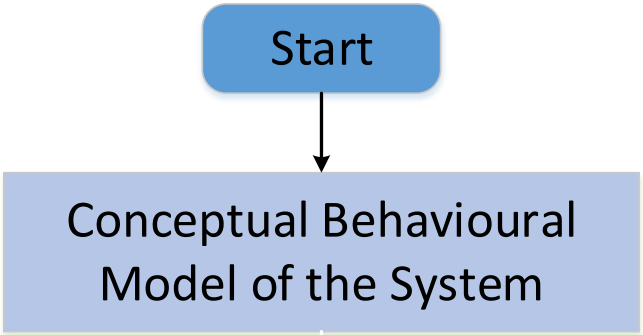


- Availability is typically derived from reliability and maintainability measures
  - A(t) = $\frac{MTBF}{MTBF + MTTR}$

    where MTBF = MTTF + MTTR
    - MTBF = Mean time between failures (Reliability Metric)
    - MTTF = Mean time to failure (Reliability Metric)
    - MTTR = Mean time to repair (Maintainability Metric)

Traditional Dependability Analysis Steps



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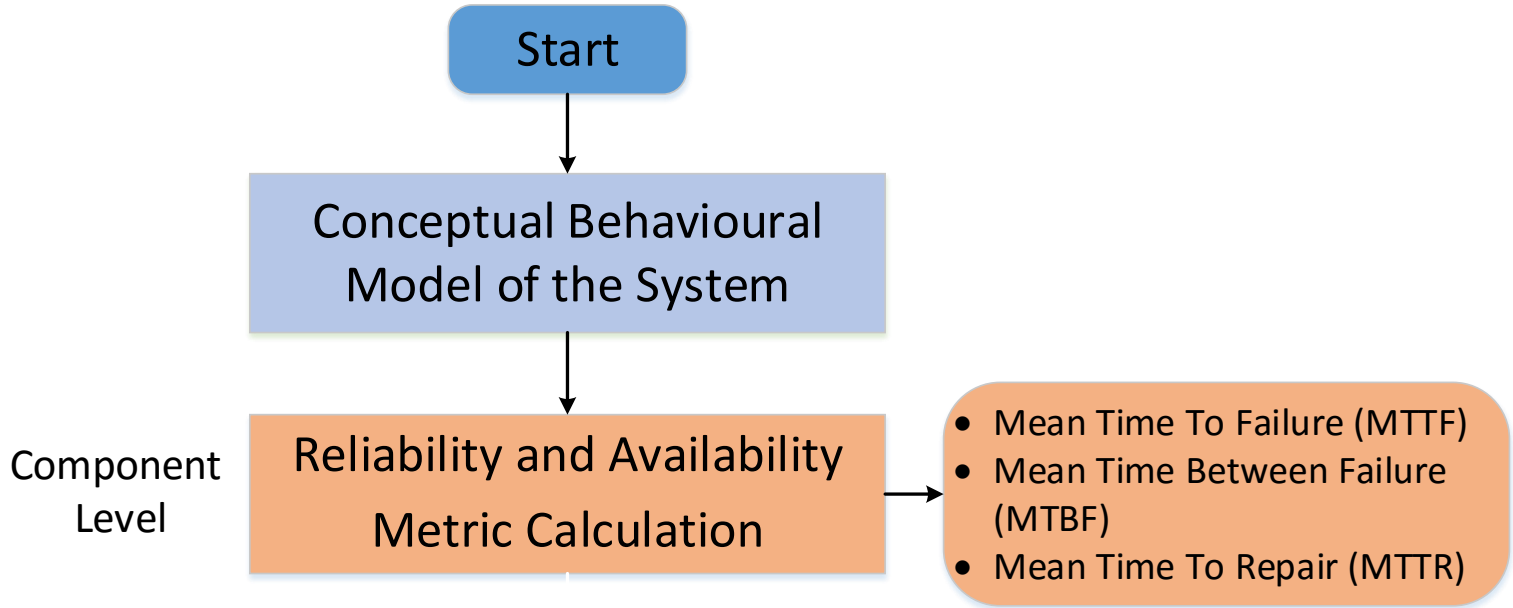

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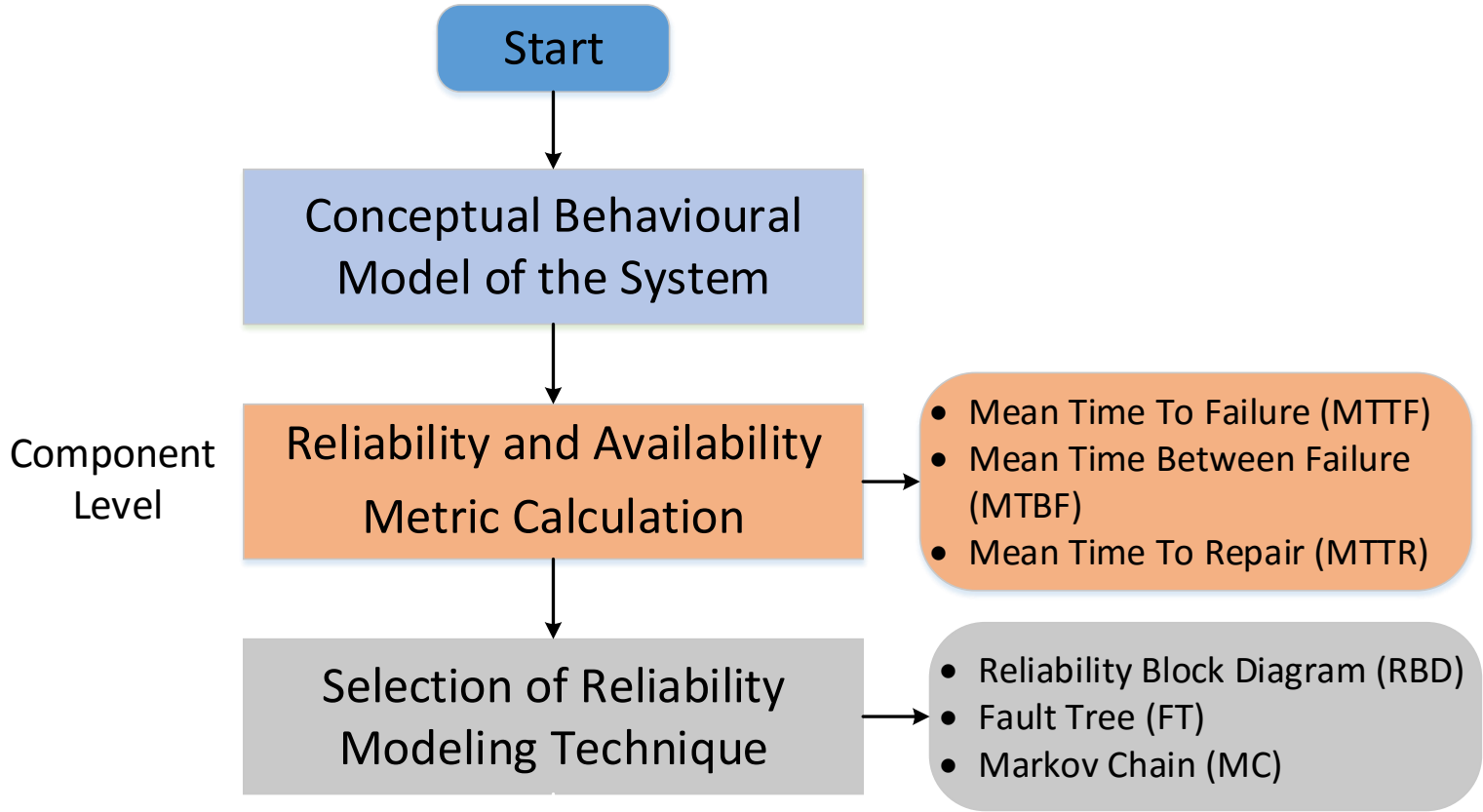

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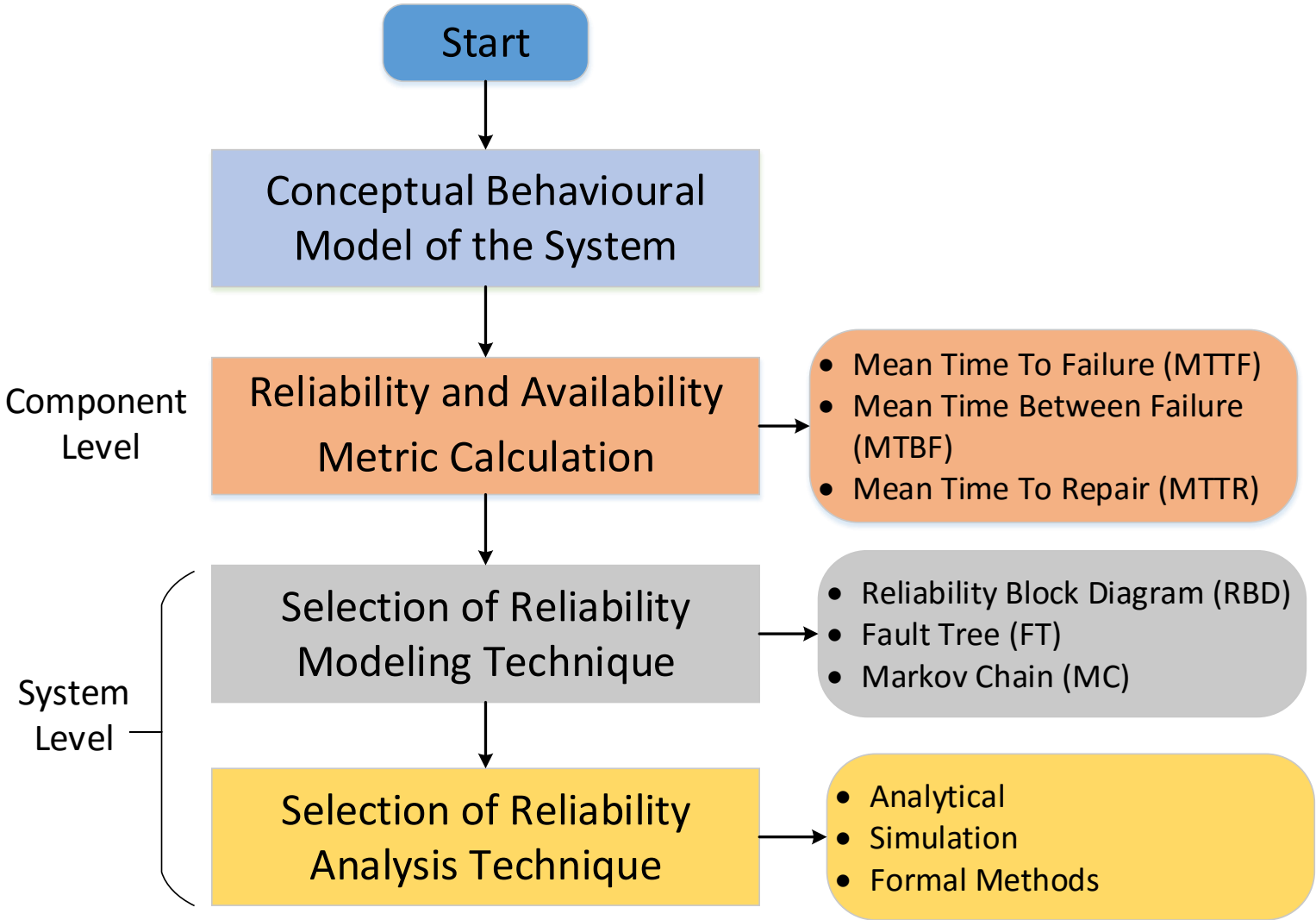


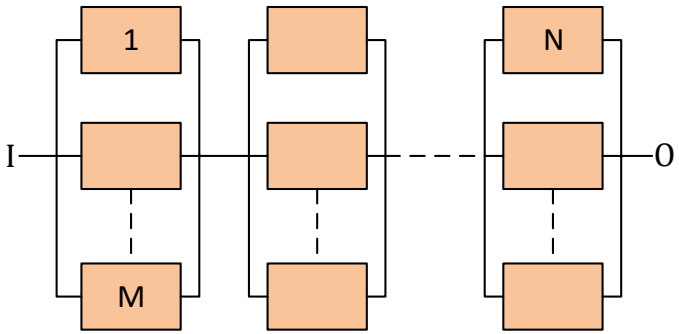
Outline

Dependability Modeling Techniques

- Some widely used modeling techniques are:
  - Reliability Block Diagram
  - Fault Tree
  - Markov Chain

Reliability Block Diagrams

- Model the failure relationship of system components as a diagram of sub-blocks and connectors (RBD)
- Judge the failure characteristics of the overall system based on the failure rates of sub-blocks



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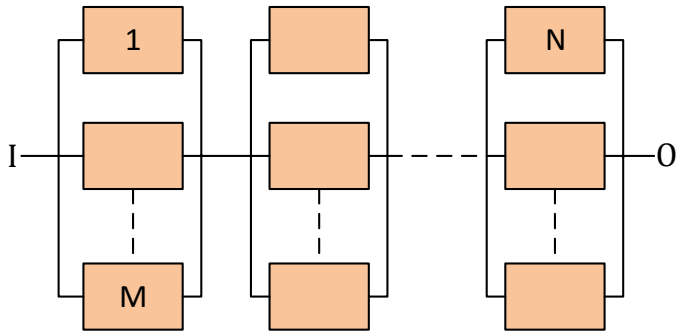
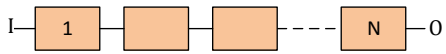
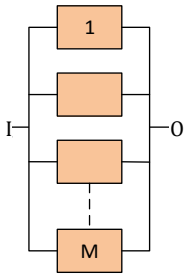
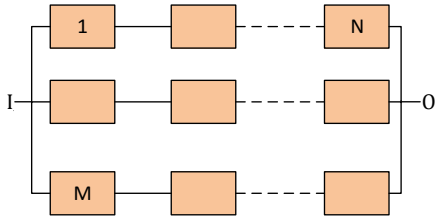
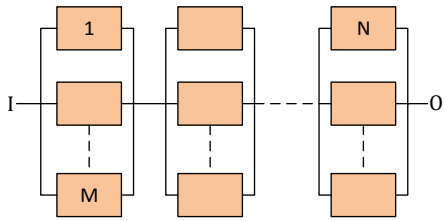
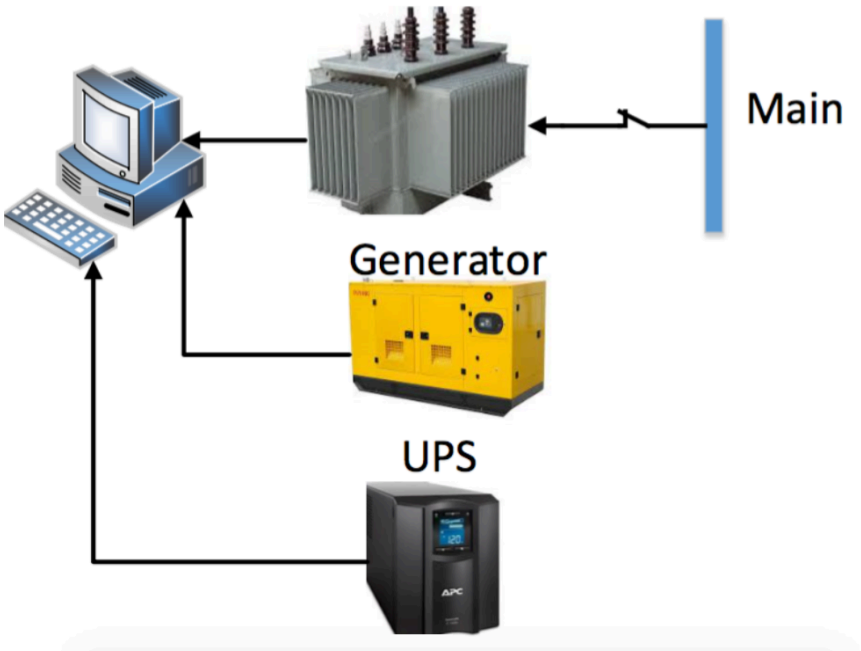


- The overall system failure happens if all the paths for successful execution fail
  - Add more parallelism to meet the dependability goals

Types of RBD

RBDs	Mathematical Expressions
	$R_{series}(t) = Pr(\bigcap_{i=1}^{N} E_i(t)) = \prod_{i=1}^{N} R_i(t)$
	$R_{parallel}(t) = Pr(\bigcup_{i=1}^{M} E_i) = 1 - \prod_{i=1}^{M}(1 - R_i(t))$
	$R_{parallel-series}(t) = Pr(\bigcup_{i=1}^{M}\bigcap_{j=1}^{N} E_{ij}(t)) = 1 - \prod_{i=1}^{M}(1 - \prod_{j=1}^{N}(R_{ij}(t)))$
	$R_{series-parallel}(t) = Pr(\bigcap_{i=1}^{N}\bigcup_{j=1}^{M} E_{ij}(t)) = \prod_{i=1}^{N}(1 - \prod_{j=1}^{M}(1 - R_{ij}(t)))$

Example: Power Supply System

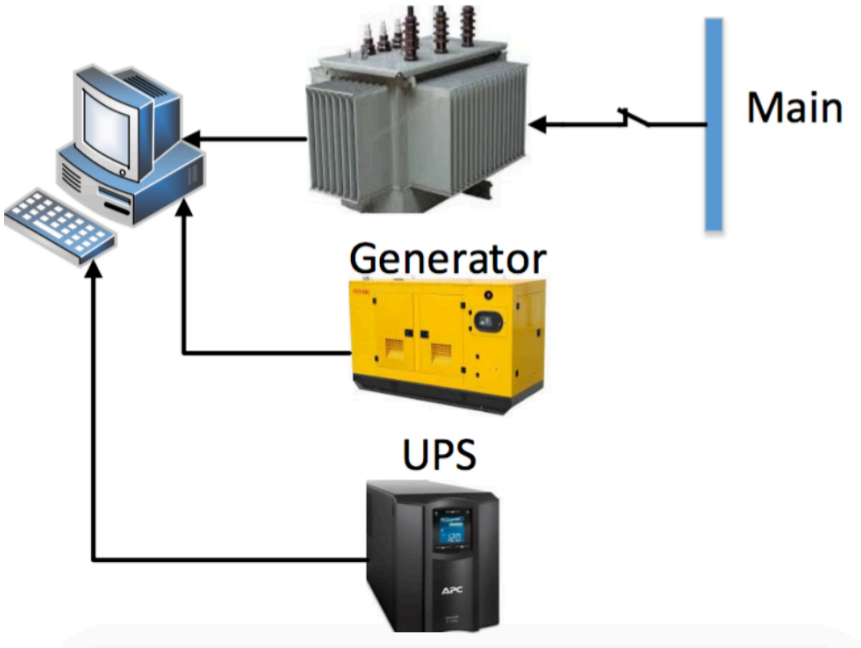


- Waqar requires continuous supply of power for his Lab PC
  - The UPS can support the load during a switch from the main supply to the generator
- Wants to determine the reliability of power supply system

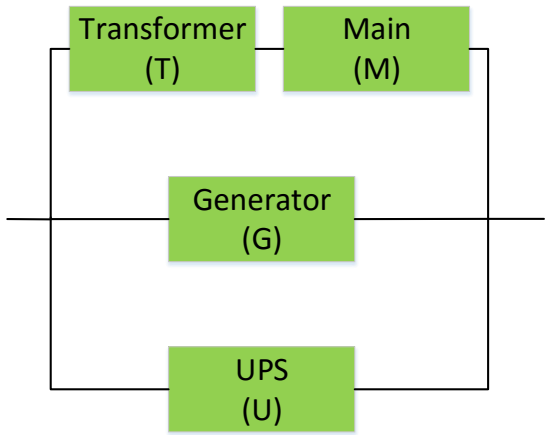
Example: Power Supply System

Step 1

Construct an RBD Model



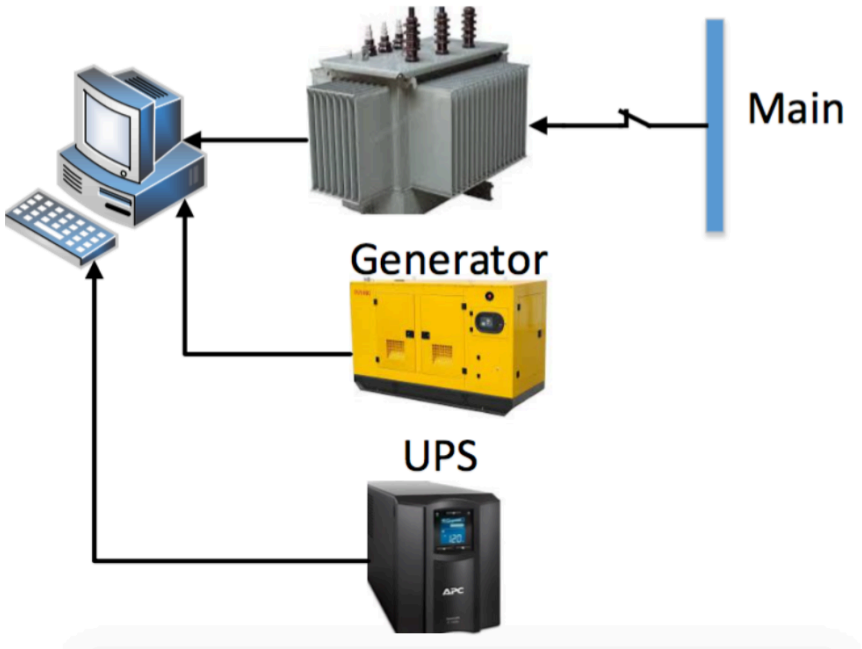
Power Supply RBD



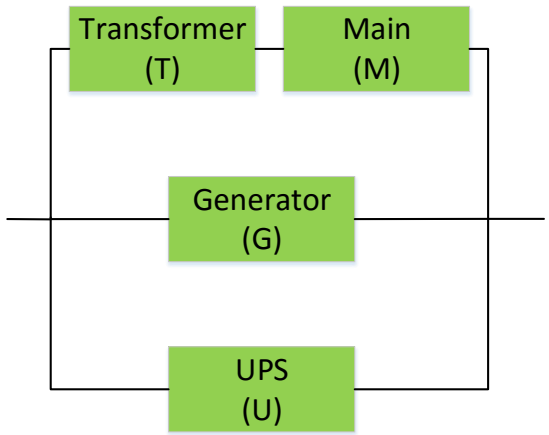
Example: Power Supply System

Step 1

Construct an RBD Model



Power Supply RBD



$$\texttt{pow\_sys\_rbd} = (\texttt{M} \cap \texttt{T}) \cup \texttt{G} \cup \texttt{U}$$

Example: Power Supply System

Step 2

Identify the RBD type

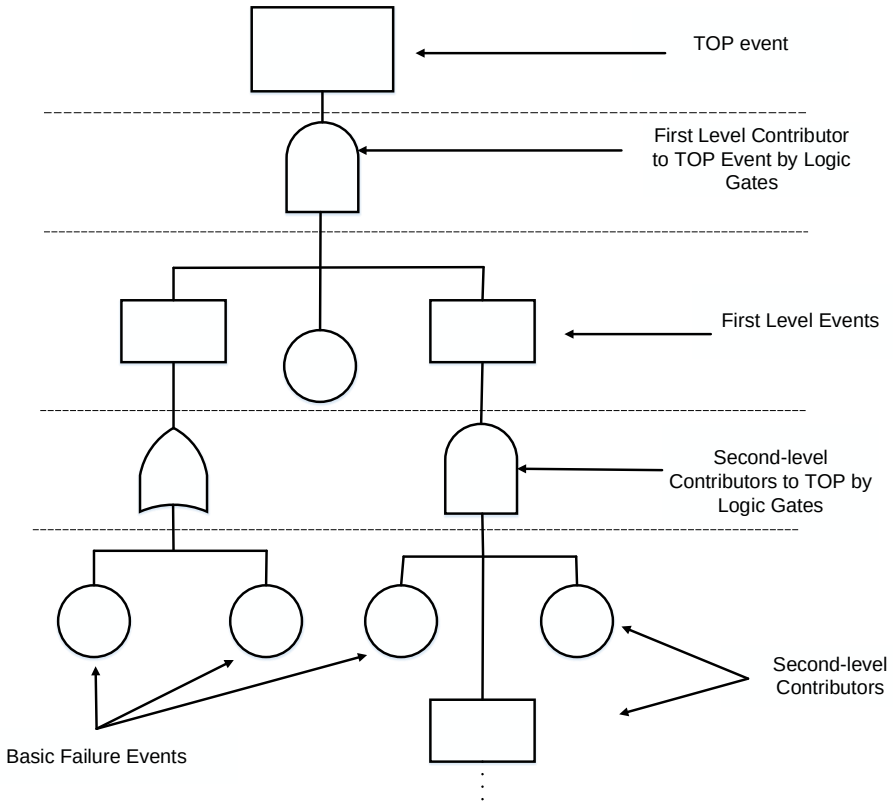
Step 3

Use the corresponding mathematical expression to evaluate the overall reliability based on the sub-components reliability

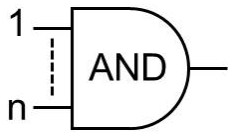
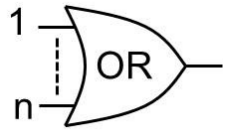
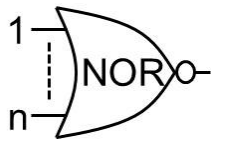
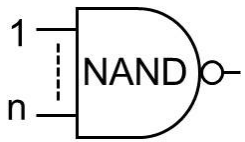
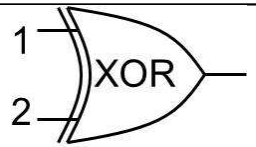
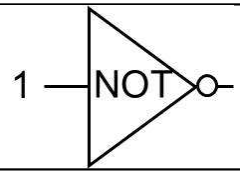
$$\mathbb{P}((\mathtt{M} \cap \mathtt{T}) \cup \mathtt{G} \cup \mathtt{U}) = 1 - (1 - \mathbb{P}(\mathtt{M}) * \mathbb{P}(\mathtt{T})) * (1 - \mathbb{P}(\mathtt{G})) * (1 - \mathbb{P}(\mathtt{U}))$$

Fault Tree

- A graphical method used to identify potential causes of system failure
- A fault tree is constructed having
  - Events: describing the failure of system components
  - Logic Gates: representing logical relationship between events
    - AND, OR, NOR, NAND, NOR etc.

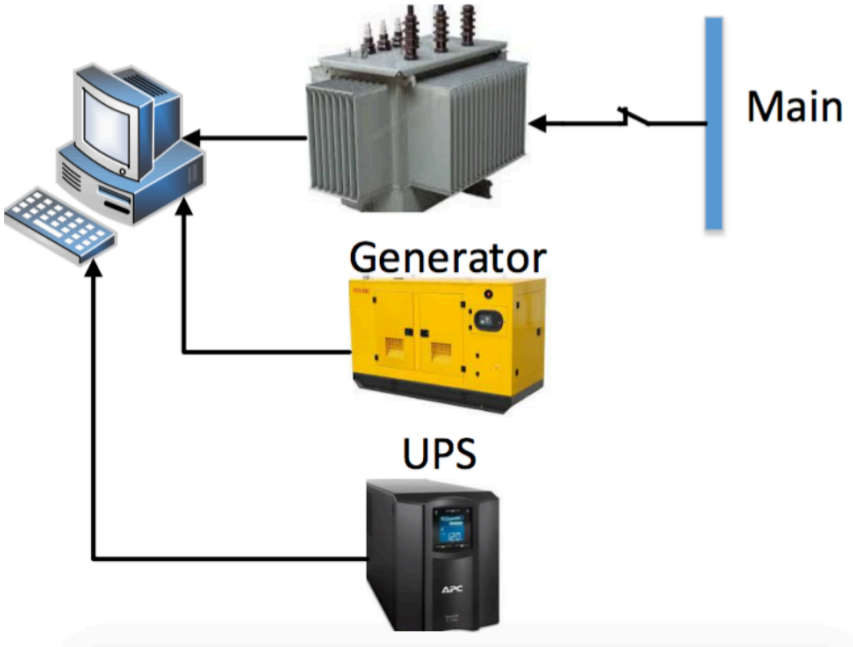


Types of FT Gates

FT Gates	Failure Probability Expressions
	$F(t) = Pr(\bigcap_{i=2}^{N} A_i(t)) = \prod_{i=2}^{N} F_i(t)$
	$F(t) = Pr(\bigcup_{i=2}^{N} A_i(t)) = 1 - \prod_{i=2}^{N}(1 - F_i(t))$
	$F(t) = 1 - F_{OR}(t) = \prod_{i=2}^{N}(1 - F_i(t))$
	$F(t) = Pr(\bigcap_{i=2}^{k} \overline{A}_i(t) \cap \bigcap_{j=k}^{N} A_i(t)) = \prod_{i=2}^{k}(1 - F_i(t)) * \prod_{j=k}^{N}(F_j(t))$
	$F(t) = Pr(\bar{A}(t)B(t) \cup A(t)\bar{B}(t))$ $= F_A(t)(1 - F_B(t)) + F_B(t)(1 - F_A(t))$
	$F(t) = Pr(\bar{A}(t)) = (1 - F_A(t))$

Example: Power Supply System

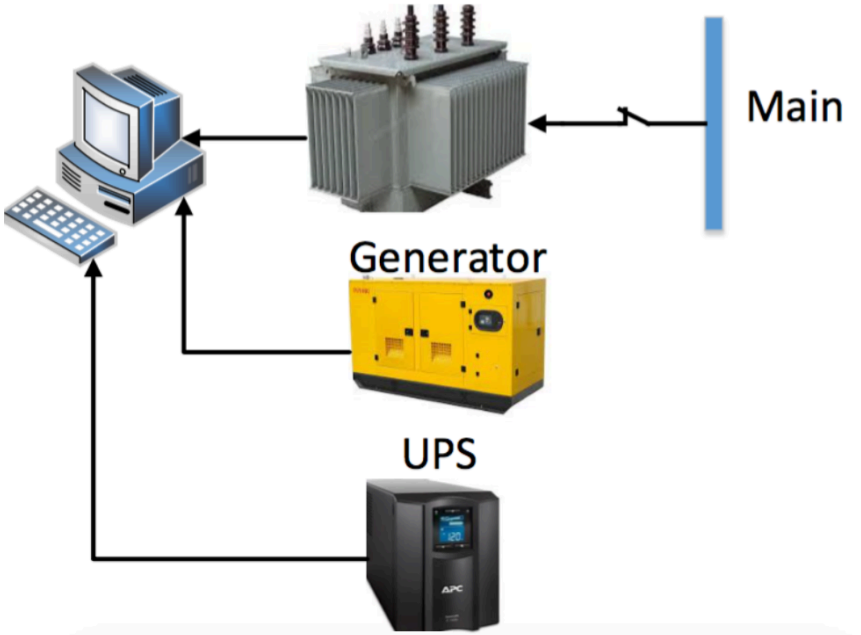
- Determine the overall failure probability?

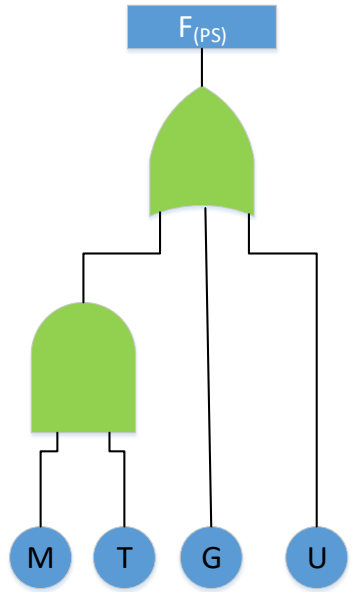


Fault Tree Analysis

Step 1

Construct a FT and represent Top Event in terms of basic events





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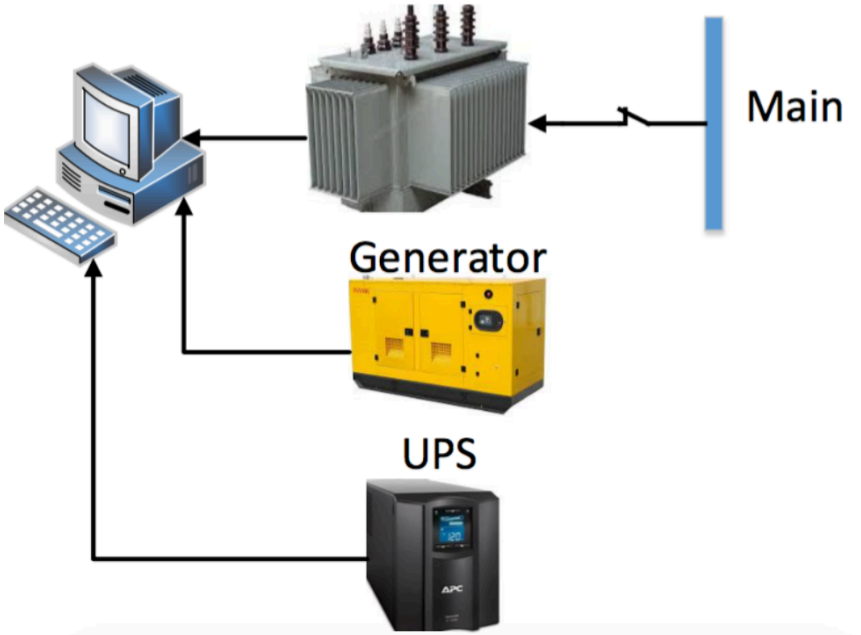
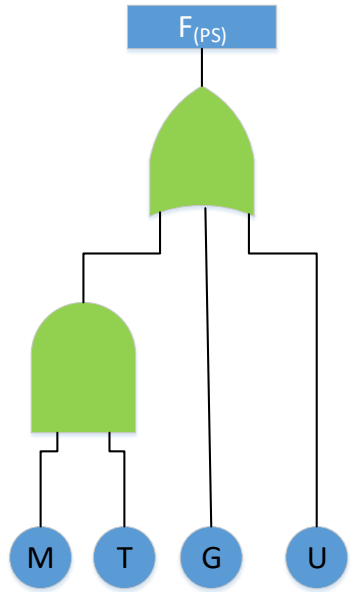




$$pow\_sys\_fail = (M \cup T) \cap G \cap U$$

Fault Tree Analysis

Step 2

Evaluate probability of failure using the Probabilistic Inclusion-Exclusion principle

$$\mathbb{P}(\bigcup_{i=1}^{n} A_i) = \sum_{J \neq \emptyset, J \subseteq \{1,2,\ldots,n\}} (-1)^{|J|-1} \mathbb{P}(\bigcap_{j \in J} A_j)$$

$$\begin{aligned}\mathbb{P}(pow\_sys\_fail) &= \mathbb{P}((M \cup T) \cap G \cap U) \\ &= \mathbb{P}(M \cap G \cap U) + \mathbb{P}(T \cap G \cap U) - \mathbb{P}(M \cap T \cap G \cap U)\end{aligned}$$

Step 3

Using Mutual Independence property

$$\mathbb{P}(pow\_sys\_fail) = \mathbb{P}(M) * \mathbb{P}(G) * \mathbb{P}(U) + \mathbb{P}(T) * \mathbb{P}(G) * \mathbb{P}(U) - \mathbb{P}(M) * \mathbb{P}(T) * \mathbb{P}(G) * \mathbb{P}(U)$$

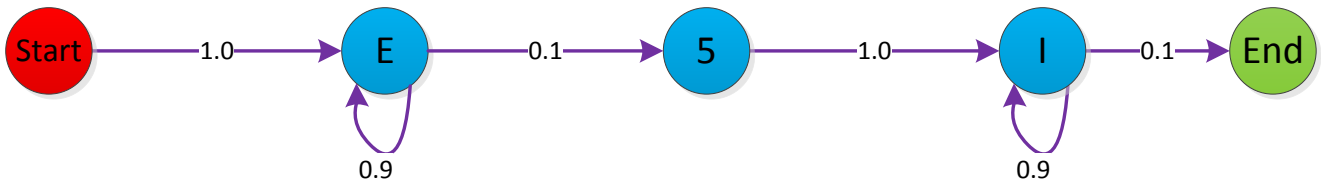
Markov Chain

- Stochastic Process

- Markov Property

Markov Chain

- Stochastic Process
  - A sequence of states
  - Determining the next state is random



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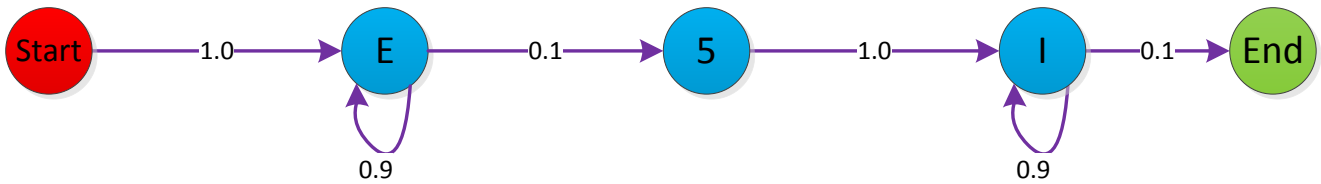
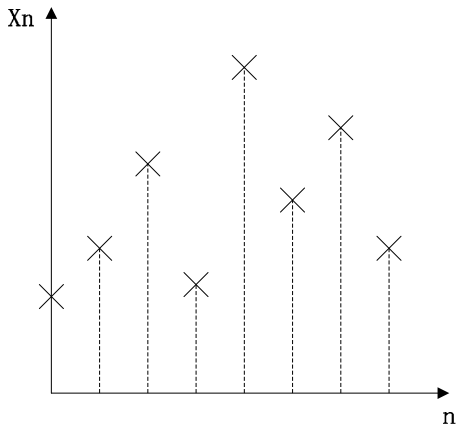


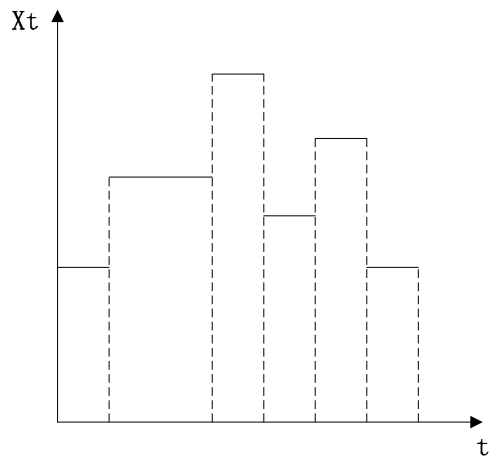
- Markov Property
  - The probability of the next state is only dependent on the current state

$$\mathcal{P}r\{X_{t_{n+1}} = f_{n+1} | X_{t_n} = f_n, \ldots, X_{t_0} = f_0\} = \mathcal{P}r\{X_{t_{n+1}} = f_{n+1} | X_{t_n} = f_n\}$$

Markov Chains - Types



Discrete-time Markov Chain



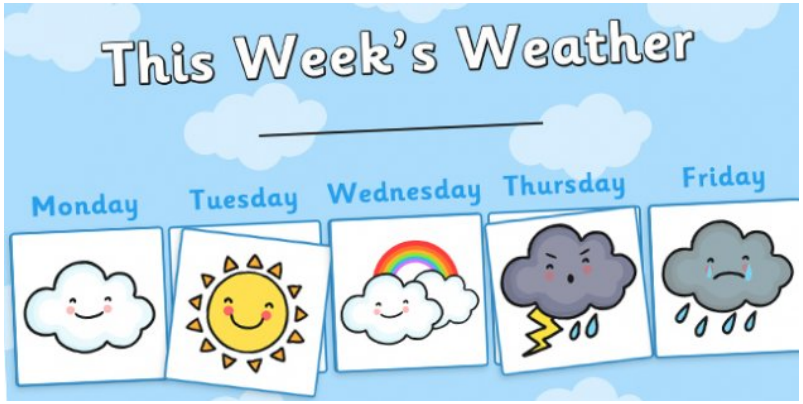
Continuous-time Markov Chain

Markov Chain - Example

- Weather Prediction Problem

Markov Chain - Example

- Weather Prediction Problem
  - Waqar records the weather conditions (sunny or rainy/snowy) daily



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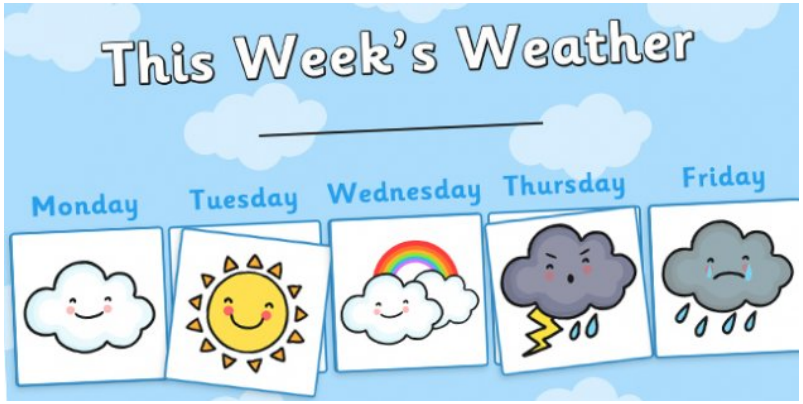
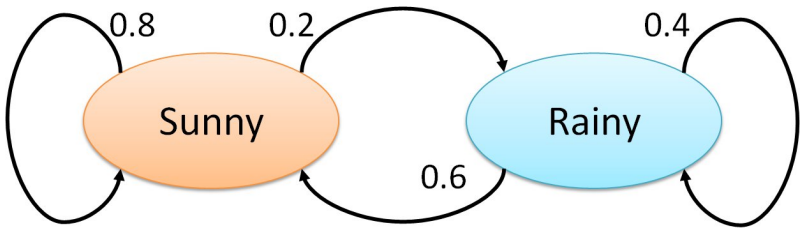


  - Based on this collected data he wants to obtain the probability of a specific weather pattern

Markov Chain - Example

- Solution: Discrete Time Markov Chains
  - Set of States = {Sunny, Rainy}
  - State Transition Probabilities can be obtained from the observed data
    - Example: P{"Tomorrow is sunny" given that "Today is sunny"}



Comparison between Dependability Modeling Techniques

Features	Reliability Block Diagram	Fault Tree	Markov Chain

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Success Paths between input and output	✓		✓

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Success Paths between input and output	✓		✓
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Success Paths between input and output	✓		✓
Failure Paths between input and output		✓	✓
Combinatorial Problems (Effect of sub-components on the failure of the whole system)	✓	✓	✓

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Combinatorial Problems (Effect of sub-components on the failure of the whole system)	✓	✓	✓
Non-combinatorial Problems (System is either inactive, failure or in standby state)			✓

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Success Paths between input and output	✓		✓
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Combinatorial Problems (Effect of sub-components on the failure of the whole system)	✓	✓	✓
Non-combinatorial Problems (System is either inactive, failure or in standby state)			✓
Large and Complex Systems	✓	✓	

Outline

Formal Dependability Analysis Techniques

Dependability models have been analyzed extensively using the following formal techniques:

- Petri Nets
- Model Checking
- Higher-order-Logic Theorem Proving

Petri Nets

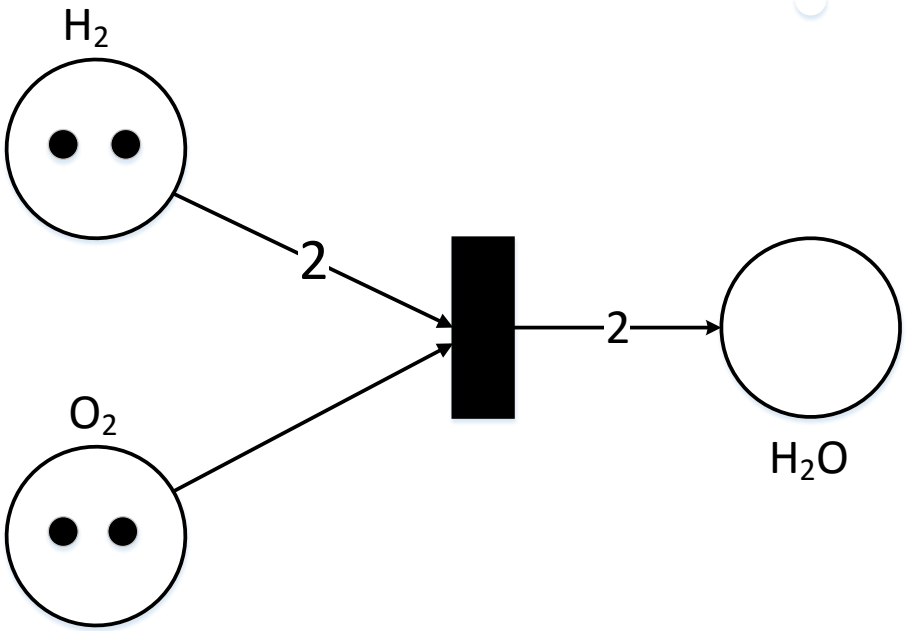
- A Petri Net is a bipartite graph consisiting of:

Places Transitions Tokens

- Transitions consume tokens from the input places and produce tokens in the output places

Example: Chemical Reaction

$$2H_2 + O_2 \Rightarrow 2H_2O$$



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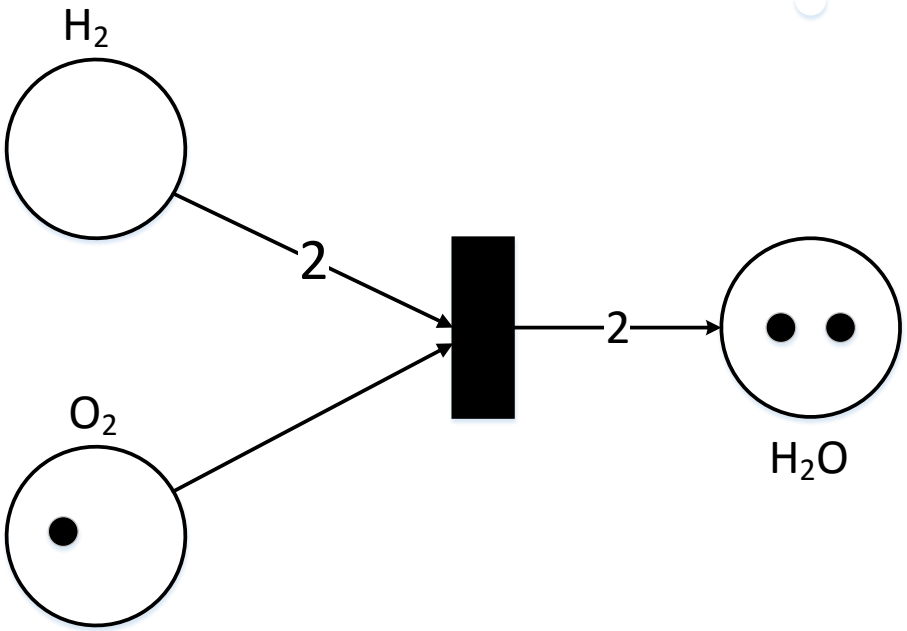

Dependability Analysis using Petri Nets:

- Colored PN (CPN) and Stochastic PN (SPN) have been extensively used for dependability analysis

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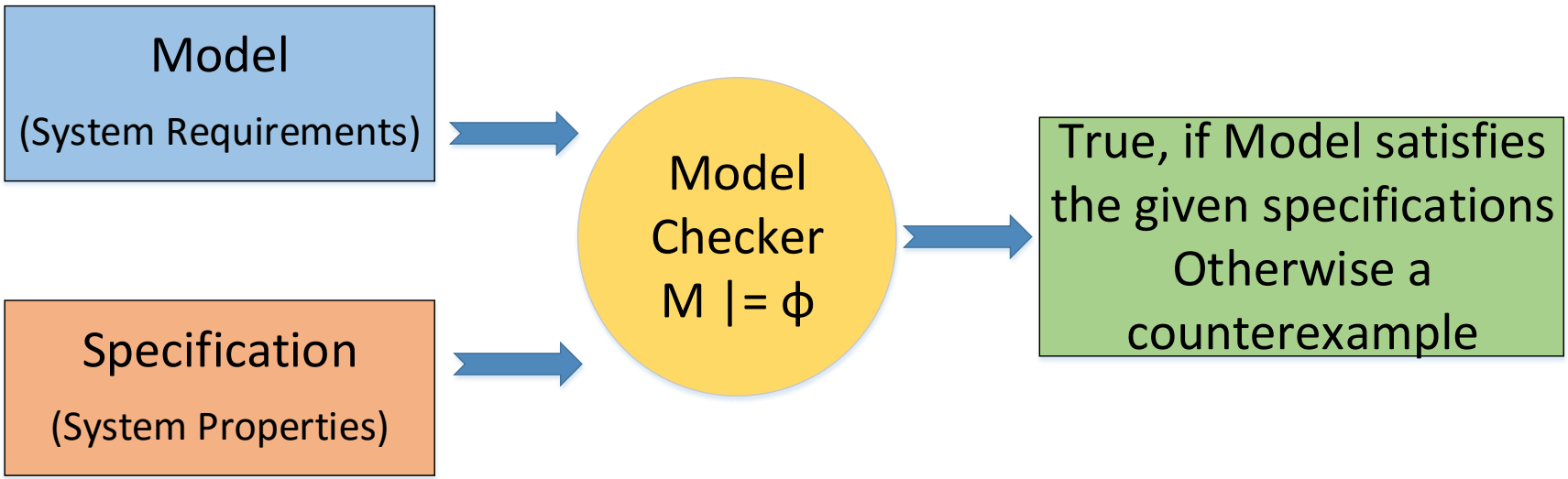
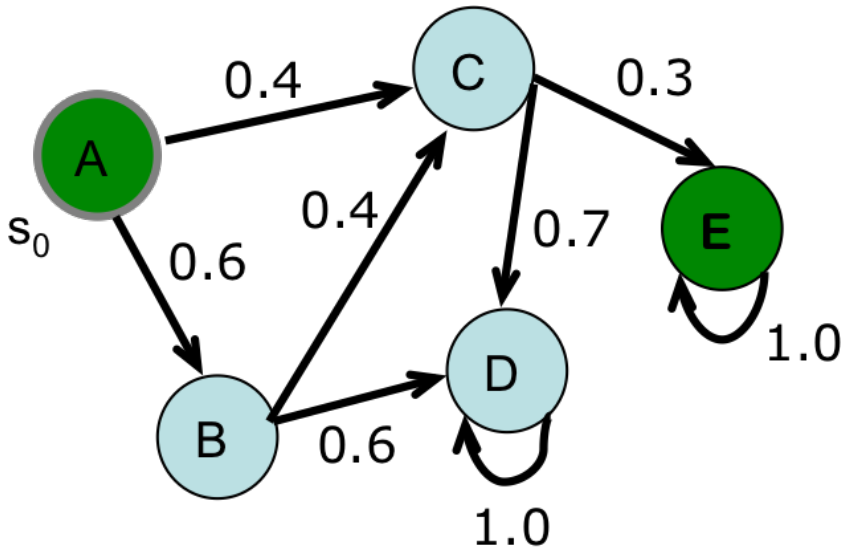
- Analyzing Markov chains with PNs
  - Client Server Queuing system (Ibe et al. TPDS-1993)
  - Fibre Distributed Data Interface (FDDI) (Christodoulou et al. ETFA-1994)
  - Low Earth Orbit (LEO) satellite (Zeng et al. JMLC-2011)

Model Checking



Probabilistic Model Checking - Example



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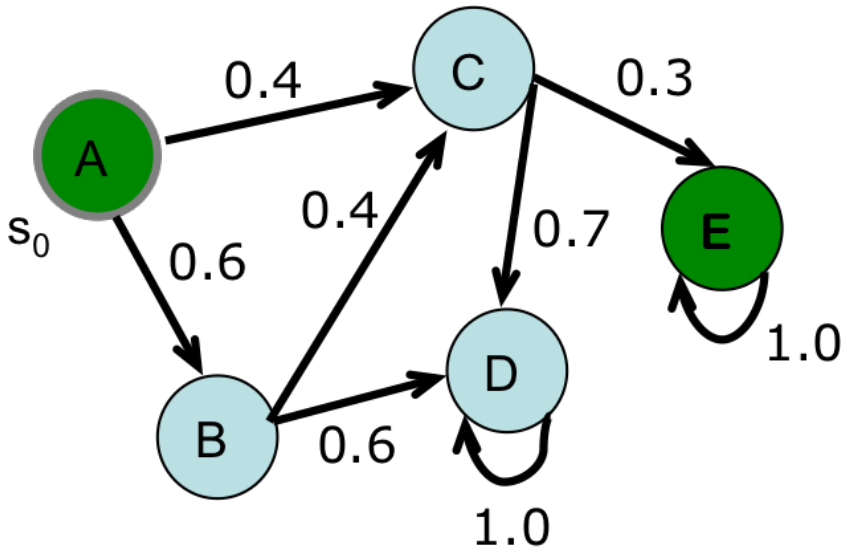

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  $0.4x0.3 + 0.6x0.4x0.3 = 0.192$

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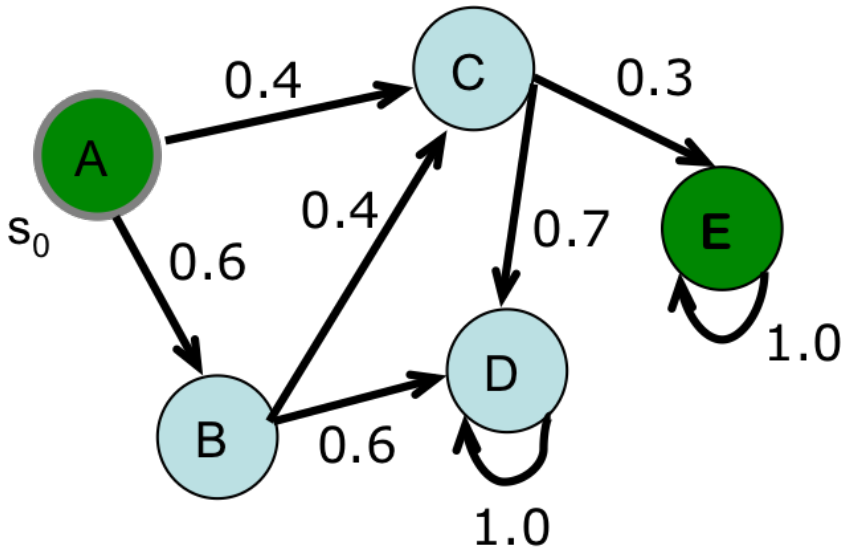

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- Probabilities associated with the validity of Temporal logic properties can be verified

Dependability Analysis using Model Checking

- Several Probabilistic and Statistical model checking tools have been used for reliability/availability assesment
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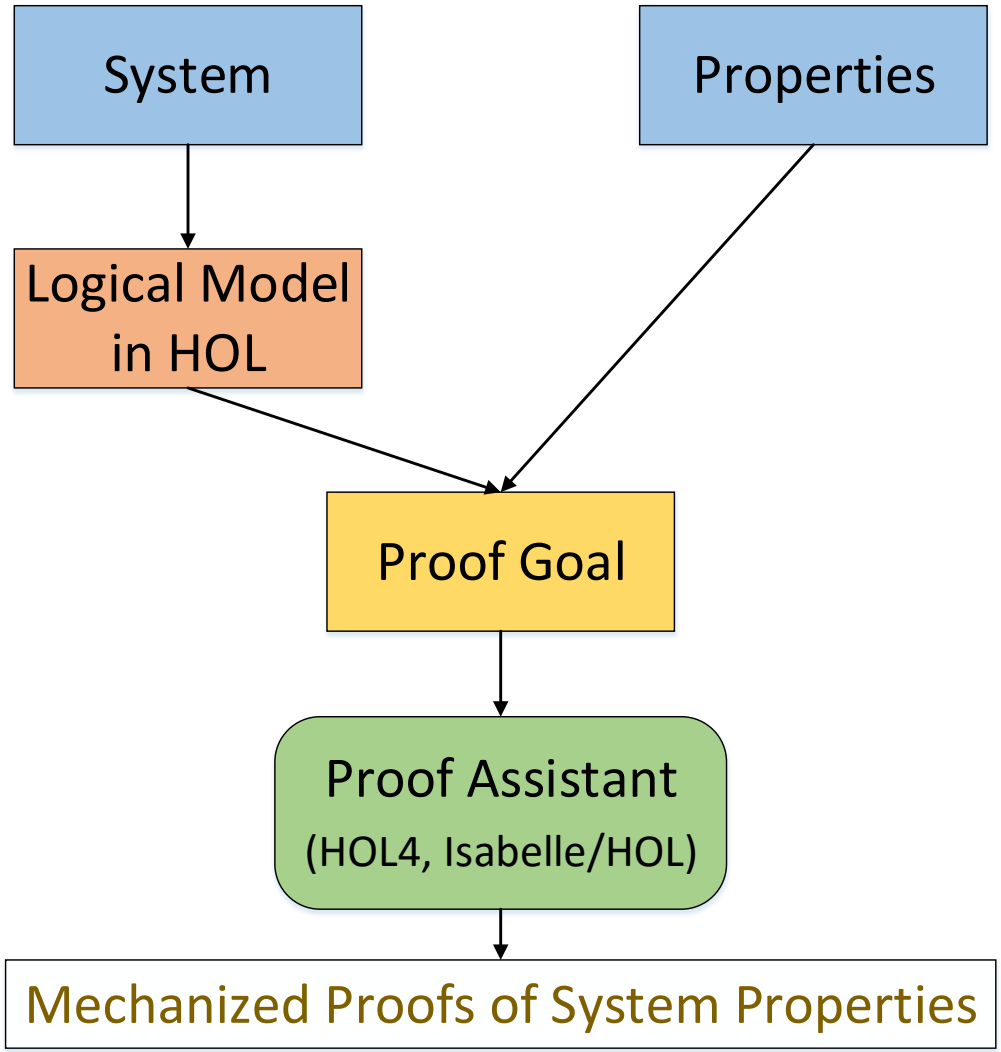
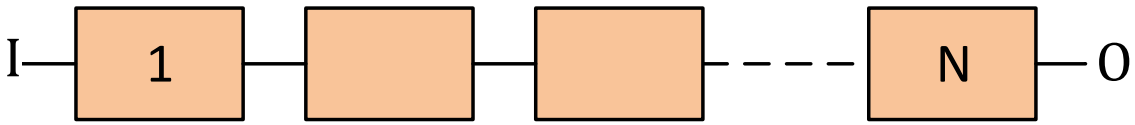
- Analysis of Real-world systems
  - Aerospace systems (Bozanno et al. SAFECOMP-2009)
  - RAID disk protocol (Gopinath et al. Tech report 2009)
  - Herschel-Planck satellite system (Pend et al. Modeling Symp. 2013)
  - Airbag system (Pend et al. Modeling Symp. 2013)
  - e-health systems used in hospitals (Pervez et al. e-HEALTHCOMM-2014)

Higher-order-Logic Theorem Proving



HOL Theorem Proving - Example: Series RBD



$$R_{series}(t) = Pr(\bigcap_{i=1}^{N} A_i(t)) = \prod_{i=1}^{N} R_i(t)$$

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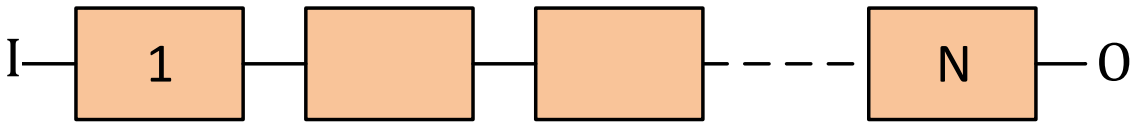

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Definition: Series RBD

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⊢ ∀ p L. series_struct p L = inter_list p L
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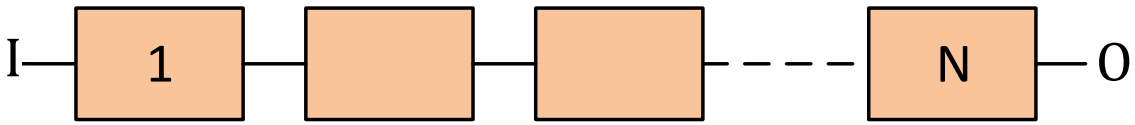

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Definition: Series RBD

$\vdash \forall$ p L. series_struct p L = inter_list p L

Theorem: Series RBD Reliability

$\vdash \forall$ p L. prob_space p $\wedge$ $\backsim$NULL L $\wedge$
mutual_indep p L $\wedge$ in_events p L $\Longrightarrow$
(prob p (series_struct p (rel_event_list p L t)) =
list_prod (list_prob p (rel_event_list p L t)))

Dependability Analysis using HOL Theorem Proving

- Probability Theory
  -  J. Hurd (2002), PhD Thesis, University of Cambridge
    Formal Verification of Probabilistic Algorithms.
  -  O. Hasan (2008), PhD Thesis, Concordia University
    Formal Probabilistic Analysis using Theorem Proving.
  -  T. Mhamdi (2011), PhD Thesis, Concorida University
    Information-Theoretic Analysis using Theorem Proving.
  -  J. Hölzl (2012), PhD thesis, Technical University of Munich
    Construction and Stochastic Applications of Measure Spaces in Higher-Order Logic.

Dependability Analysis using HOL Theorem Proving

- Dependability Analysis of a Component
  - Reconfigurable Memory Arrays (Hasan et al. TC-2010)
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- Dependability Analysis using RBDs and FTs
  - Oil and Gas Pipelines (Waqar et al. CICM-2014)
  - WSN Transport Protocols (Waqar et al. WiMob-2015)
  - Logistic Supply Chain (Waqar et al. IWIL-2015)
  - Satellite Solar Array (Waqar et al. CICM-2015)

Comparison between Dependability Analysis Techniques

Feature	Paper-and-pencil Proof	Simulation Tools	Petri Nets	Theorem Proving	Model Checking

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Outline

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Future Directions:

- Analysis of Dynamic RBDs and FTs
- Using Theorem Proving to conduct Markov Chains based dependability analysis
  - Foundational Support is available in HOL4 (L. Liu et al., ATVA-2011) and Isabelle/HOL (J. Hölzl et al., TACAS 2012)

Thanks!

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