Interactive Simplifier Tracing and Debugging in Isabelle

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Agenda

Isabelle

- interactive proof assistant
- powerful automation
  - classical and equational reasoning
  - decision procedures (e.g. linear arithmetic)
  - integration with external automated theorem provers
  - ...
- IDE with continuous proof checking based on jEdit

Simplifier

- one of the core tactics in Isabelle
- huge: more than 1800 lines of code
- applies rewrite rules to terms
- rules can be conditional: preconditions solved recursively
- rules can be lazy: “simprocs” can generate rules on the fly
- goals can be conditional: local assumptions are used

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Simplifier

Example: Conditional rewrite rules

$\mathbf{x}, \mathbf{y} \in \mathbb{N}$

$$2 \cdot x = x + x \tag{1}$$

$$x < y \implies x - y = 0 \tag{2}$$

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$$0 < x \implies 0 < y \implies 0 < x + y \tag{4}$$

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- unexpected result
- non-termination

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tackled by tracing

Simplifier Trace

Lists all rewriting steps, but:

- potentially huge
- can't be filtered (e.g. "trace only applications of X and Y")
- offers no hierachical structure
- problematic with non-termination

```
[1]SIMPLIFIER INVOKED ON THE FOLLOWING TERM:
10 ^ 5 * 10 ^ 5 = 10 ^ 10
[1]Applying instance of rewrite rule "Power.semiring_numeral_class.power_numeral"
numeral ?k1 ^ numeral ?l1 ≡ numeral (Num.pow ?k1 ?l1)
[1]Rewriting:
10 ^ 5 ≡ numeral (Num.pow (num.Bit0 (num.Bit1 (num.Bit0 num.One))) (num.Bit1 (nu
[1]Applying instance of rewrite rule "Num.pow.simps_3":
Num.pow ?x1 (num.Bit1 ?y1) ≡ Num.sqr (Num.pow ?x1 ?y1) * ?x1
[1]Rewriting:
Num.pow (num.Bit0 (num.Bit1 (num.Bit0 num.One))) (num.Bit1 (num.Bit0 num.One)) ≡
Num.sqr (Num.pow (num.Bit0 (num.Bit1 (num.Bit0 num.One))) (num.Bit0 num.One)) * r
[1]Applying instance of rewrite rule "Num.pow.simps_2":
Num.pow ?x1 (num.Bit0 ?y1) ≡ Num.sqr (Num.pow ?x1 ?y1)
[1]Rewriting:
Num.pow (num.Bit0 (num.Bit1 (num.Bit0 num.One))) (num.Bit0 num.One) ≡ Num.sqr (N
[1]Applying instance of rewrite rule "Num.pow.simps_1":
Num.pow ?y num.One ≡ ?y
[1]Rewriting:
Num.pow (num.Bit0 (num.Bit1 (num.Bit0 num.One))) num.One ≡ num.Bit0 (num.Bit1 (n
```

Agenda

Overview

- interactive
- breakpoints on terms and theorems
- configurable verbosity
- integrated into Isabelle/jEdit

Demonstration

Related Work

SWI-Prolog

- offers interactive tracing
- supports breakpoints
- speciality: marking goals as success

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  - In Isabelle: difficult because of proof kernel

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- supports breakpoints
- speciality: during rewriting, issue new goal
  - In Isabelle: rarely needed because of parallel processing

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Challenges

Selective Memory Clearing

Scenario

1. rewrite step fails
2. user chooses to redo the step
3. simplification starts anew
4. memoization kicks in, step fails again

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Challenges

Context Handling

- simplification result depends on local assumptions
- memoization might not make sense across different contexts

$$(P \Longrightarrow P) \Longrightarrow (Q \Longrightarrow P) \Longrightarrow R$$

Challenges

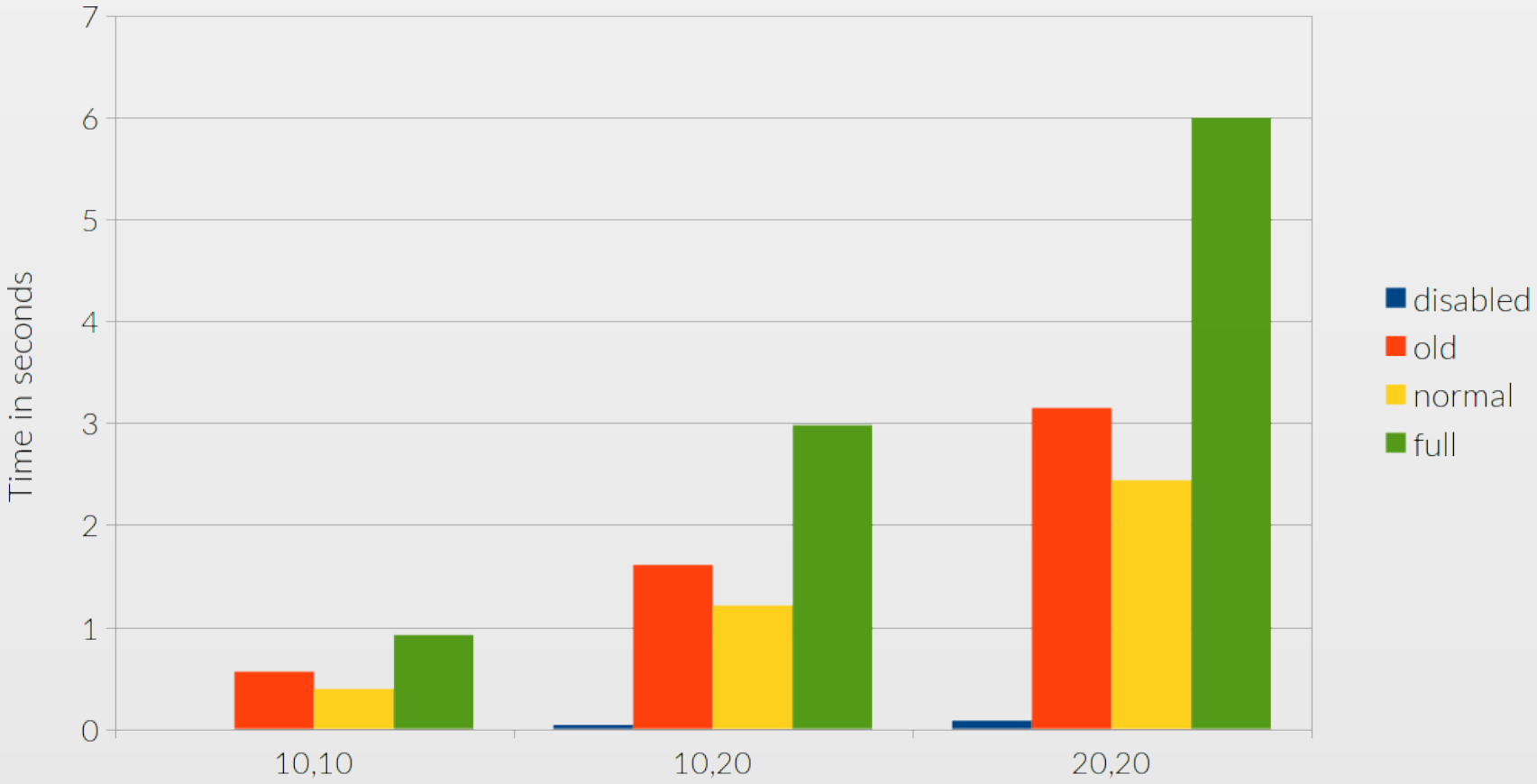
User Experience

- user feedback is generally positive
- used for detecting erratic rules, analyzing simplifier runtime, …
- very flexible, but: every additional option generates confusion

Agenda

Performance

Simplifying $10^x \cdot 10^y$



Test machine: Core i7, 3.7 GHz

A Parallelized Simplifier?

- tracing is completely asynchronous
- supports multiple questions at the same time
- but: unused by the simplifier
- proof of concept: development of a tiny, parallel simplifier

A Parallelized Simplifier?

Lessons Learned

Advantages

- almost trivial to implement for a toy simplifier
- GUI part works out of the box

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Disadvantages

- potentially confusing for users
  - lots of spurious messages
  - better filtering required?
  - holding back messages required?

Outcomes

- a *generic* tracing facility
  - using its interface requires little changes to a tactic
  - parallelization-ready
  - but not 100% there yet
- first steps towards instrumenting the simplifier
  - Should all tactics be written in continuation-passing style?

Future Work

- support for more tactics
- support for other traces (unifier, simp debug, …)
- memoization: fuzzy matching
- term provenance (“Where does that ‘5’ come from?”)
- tighter integration into Isabelle/jEdit

Q & A