Hipster: Integrating Theory Exploration in a Proof Assistant

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Introduction: Theory Exploration

Theory Exploration Paradigm [Buchberger-2000]:

- Theorems not proved in isolation.
- Rather, explore whole theories:
  - Prove routine lemmas.
  - Proceed to more complex theorems.
  - Possibly backtrack and prove more lemmas.
  - New theories on top of old ones.
- Interactive theorem proving:
  - Creative/hard steps left to user.

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Our work:

Automatically discover **new** and **interesting** lemmas in inductive theories.

Inductive Theorem Proving and Theory Exploration

Example Domain: Proofs by induction

- Often need lemmas (also needing induction).
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Background: HipSpec

- Inductive prover for Haskell.
- Generate (equational) conjectures. Tested, not proved.
- Apply induction, then call off the shelf FO-provers.

Hipster: Theory Exploration for Isabelle/HOL

- Translate Isabelle/HOL theory to Haskell.
- Use conjecture generation from HipSpec.
- Currently only equational conjectures.
- Prove in Isabelle (LCF-style).
  - Keep interesting theorems (need induction).
  - Discard if trivial proof.

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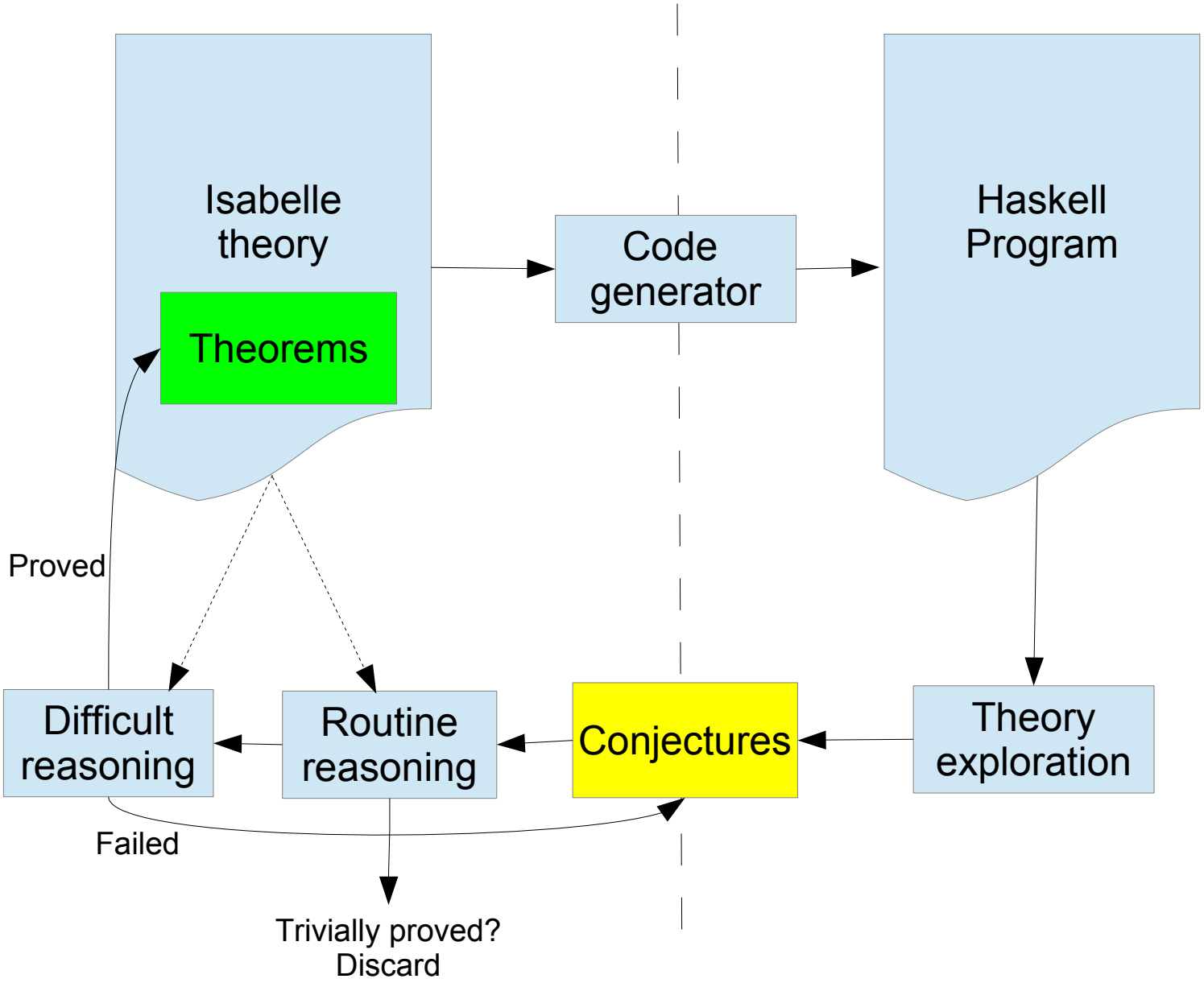
Demo: Exploring a theory about binary trees

Hipster: Overview



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- Set of *functions* and *variables*.
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Ongoing and Further Work

- Experiments with different tactics for hard/routine reasoning.
- **Conditional lemmas:**
  - Given a side condition, generate lemmas.
  - E.g. `sorted(xs) ==> sorted(insert x xs)`

Conclusion

- Automatically find and prove routine lemmas.
- LCF-style re-checkable proofs.
- Incremental exploration, store lemmas in libraries.
- User can control search space.
- Lemmas enhance automated tactics, e.g. Sledgehammer.