



Interactive Theorem Provers (ITPs)

...are programming languages applied for:

- ★ **software verification in industry:**
 - ARM microprocessor: 20,000 lines of code;
 - verified C compiler: 50,000 lines of code;
 - seL4 microkernel: 200,000 lines of code.
- ★ **and formalisation of mathematics:**
 - Four Colour theorem: 60,000 lines of code;
 - Feit-Thompson theorem: 170,000 lines;
 - Flyspeck project: 325,000 lines of code.

Challenges

- Manual handling of various proofs, strategies and libraries becomes difficult;
- Team-development is hard, especially since ITPs are sensitive to user notation;
- Comparison of proofs and proof similarities across libraries or in one big library is hard.

Our solution: ML4PG

Proof General [1] is a user interface for several existing ITPs. **ML4PG** [4] is a machine-learning extension to **Proof General** [1] that:

- finds common proof-patterns in proofs across various scripts, libraries, users and notations;
- provides proof-hints, especially in industrial cases where routine similar cases are frequent, and effort is distributed across several programmers.

ML4PG session for Coq/SSReflect

```

1 subgoals, subgoal 1 (ID 68)
| 0 => a
| S p => helper_fact p (n * a)
end.

Definition fn_fact (n : nat) := helper_fact n 1.

(* 3. We prove that the algorithm satisfies the spec, by
proving first that the
helper is appropriately related to theta and then t
*hat fn is theta on ok
inputs. *)

Lemma helper_fact_is_theta n a :
  helper_fact n a = a * theta_fact n.
Proof.
move : a; elim : n => [a] n IH a /=.
by rewrite /theta_fact fact0 mulIn1.
by rewrite IH /theta_fact fact5 mulInA [a * _] mulInC.
Qed.

Lemma fn_fact_is_theta n : fn_fact n = theta_fact n.
Proof.
rewrite /fn_fact.[]

```

Step 1: Feature extraction

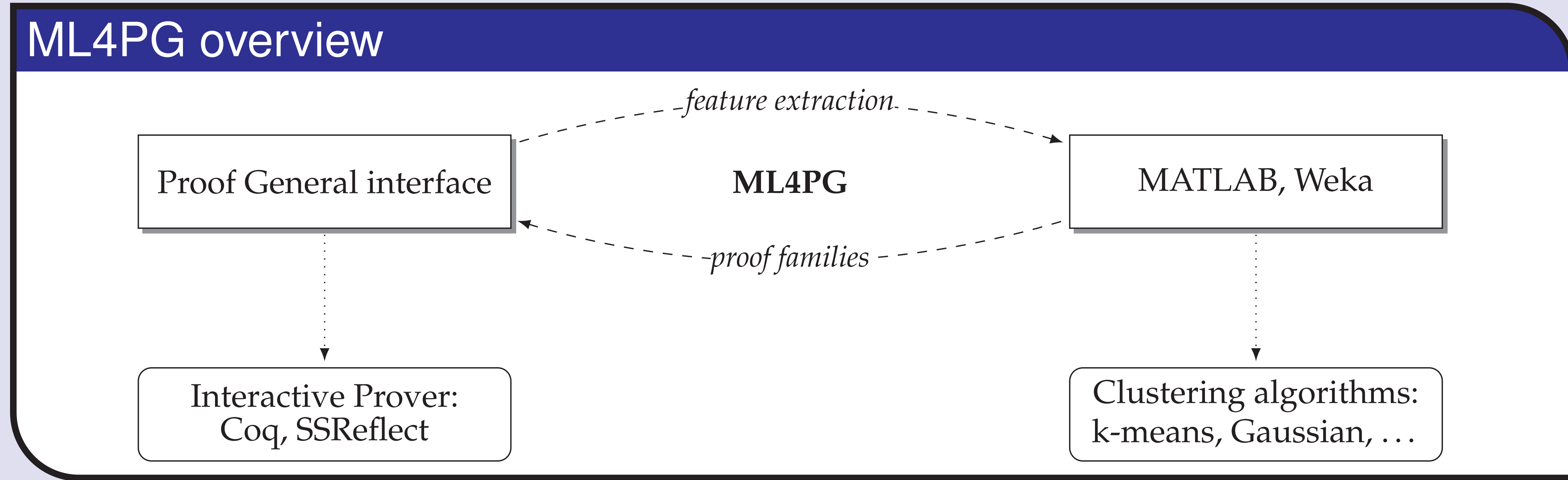
- ML4PG works on the background of Proof General, and extracts statistical features from interactive proofs in Coq [2] and SSReflect [3];
- The features reflect shapes of lemmas, structure of proofs, and patterns of user interaction with the ITP.
- Proof trace method captures statistical relation between several proof steps.

Step 2: Machine learning tools

- As higher-order proofs in general can take an infinite variety of shapes and sizes, ML4PG does not use any a priori given training labels.
- it uses unsupervised learning (clustering) algorithms implemented in MATLAB and Weka; and allows the user to adjust learning parameters, e.g. the size and proximity of clusters.
- The output shows families of related proofs.

Step 3: Interaction with ML4PG

- ML4PG automatically sends the gathered statistics to a chosen machine-learning interface and triggers execution of a clustering algorithm of the user's choice;
- it does some gentle post-processing of the results given by the machine-learning tool, and displays families of related proofs to the user.



Discovered Proof Families

Proof hints provided by ML4PG for the proof of **Lemma 1**: all proof families below contain proofs of already proven *, **, and ***.

| Clustering algorithm: | g = 1 | g = 2 | g = 3 |
|-------------------------|-----------|----------|----------|
| Gaussian | 9 | 0 | 0 |
| K-means (MATLAB) | 20 | 3 | 3 |
| K-means (Weka) | 29 | 10 | 2 |
| FarthestFirst | 27 | 24 | 0 |

Size of dataset: ≈ 150 lemmas. The granularity parameter g ranges from 1 (producing big and general clusters) to 5 (producing small and precise clusters).

Case Study: Verification of Java Virtual Machine with ML4PG

Java Virtual Machine (JVM) is a stack-based abstract machine which can execute Java bytecode. We modelled a subset of the JVM in Coq, verifying the interpreter for JVM programs. This work is inspired by the ACL2 proofs about JVMs [5].

```

0 : iconst 1
1 : istore 1
2 : iload 0
3 : ifeq 13
4 : iload 1
5 : iload 0
6 : imul
7 : istore 1
8 : iload 0
9 : iconst 1
10 : isub
11 : istore 0
12 : goto 2
13 : iload 1
14 : ireturn

static int factorial(int n)
{
  int a = 1;
  while (n != 0){
    a = a * n;
    n = n-1;
  }
  return a;
}

```

JVM model:

```

counter: 0
stack:
local variables:

```

```

Fixpoint helper_fact (n a : nat) :=
  match n with
  | 0 => a
  | S p => helper_fact p (n * a)
end.

Definition fn_fact (n : nat) :=
  helper_fact n 1.

```

Benefits of ML4PG

- ... can be switched on/off on user's demand;
- ... does not assume any machine-learning knowledge from the user;
- ... allows the user to make choices regarding approach to levels of proofs, size and granularity of clusters, and particular statistical algorithms;
- ... tolerant to mixing and matching different proof libraries and different notation used in proofs across different users.

Case study: ML4PG Role in Proof Pattern Discovery

As part of JVM verification process, we needed to prove in Coq the following lemma:

Lemma 1 (Factorial JVM lemma) $\forall n \in \mathbb{N}$, running the bytecode associated with the factorial program with n as input, the Coq JVM produces a state which contains $n!$ on top of the stack.

After processing the proof statistics of 150 lemmas in the library, ML4PG correctly suggested to reuse the proof strategy from similar (already proven) lemmas concerning different operations:

- * multiplication JVM lemma
- ** power JVM lemma
- *** exponentiation JVM lemma

References

- [1] D. Aspinall. *Proof General: A Generic Tool for Proof Development*. In TACAS'00, LNCS 1785, pp. 38–43. 2000.
- [2] Y. Bertot and P. Castéran. *Coq'Art: the Calculus of Constructions*. Springer-Verlag, 2004.
- [3] G. Gonthier and A. Mahboubi. *An introduction to small scale reflection*. J. of Formalized Reasoning, 3(2):95–152. 2010.
- [4] E. Komendantskaya, J. Heras, and G. Grov. *Machine learning in Proof General: interfacing interfaces*. To be published in EPTCS Post-proceedings of UITP'12. 2013.
- [5] J.S. Moore. *Models, Algebras and Logic of Engineering Software*, chapter Proving Theorems about Java and the JVM with ACL2, pages 227–290. IOS Press, 2004.